NUMERICAL MODELING OF TWO-DIMENSIONAL CONFINED FLOWS

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PREPARED FOR NASA LEWIS RESEARCH CENTER

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ABSTRACT

A new approach to numerically model two-dimensional confined flows is presented. The flow in the duct is partitioned into finite streams. The difference equations are then obtained by applying conservation principles directly to the individual streams. Also presented is a listing of a computer code based on this approach in FORTRAN IV language. The code computes two-dimensional compressible turbulent flows in ducts when the duct area along the flow is specified and the pressure gradient is unknown.

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1. INTRODUCTION

Two-dimensional effects, such as the growth of momentum and thermal boundary layers, in flows through nozzles, channels, and diffusers have been traditionally studied by separating the flow into a core and a boundary layer.[1] The core flow is then calculated by one-dimensional potential flow equations and the boundary layer by momentum integral approach. Some flows cannot be accurately calculated with this approach. For example, in flows with large differences between the core and the wall temperatures substantial heat flow in or out of the core weakens the assumption of a radially uniform core. Again in calculating performance of diffusers with high inlet blockage factor, as in MHD diffusers, the flow cannot be meaningfully separated into a core and a boundary layer.

Recently many approaches, based on finite difference methods, have been developed to calculate confined flows without the separation into core and boundary layer. These approaches can be broadly divided into two classes. In one class the finite difference equations are obtained by differencing the flow differential equations either in their primitive form or in some transformed form. In the other class the finite difference equations are obtained directly from the conservation principles. Examples of these two classes, along with their interrelationship, are given in Fig. 1.

This report deals with the approach of Ref. 7--shown bordered by heavy line in Fig. 1. The original paper describing this approach is

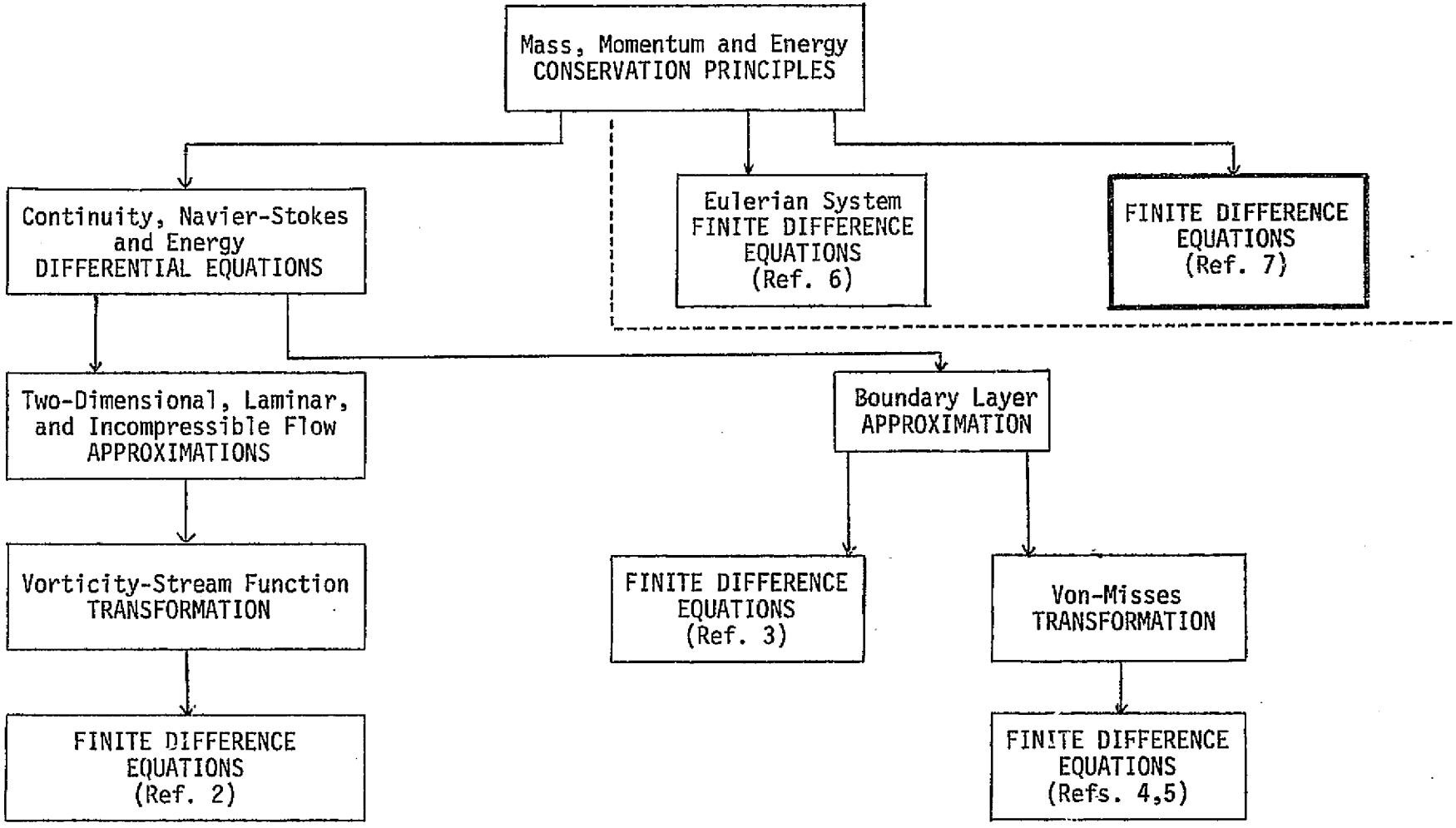
Mass, Momentum and Energy
CONSERVATION PRINCIPLES
Continuity, Navier-Stokes
and Energy
DIFFERENTIAL EQUATIONS
Eulerian System
FINITE DIFFERENCE
EQUATIONS
(Ref. 6)
FINITE DIFFERENCE
EQUATIONS
(Ref. 7)
Two-Dimensional, Laminar,
and Incompressible Flow
APPROXIMATIONS
Boundary Layer
APPROXIMATION
Vorticity-Stream Function
TRANSFORMATION
FINITE DIFFERENCE
EQUATIONS
(Ref. 3)
Von-Misses
TRANSFORMATION
FINITE DIFFERENCE
EQUATIONS
(Ref. 2)
FINITE DIFFERENCE
EQUATIONS
(Refs. 4,5)

Figure 1

included in Appendix B. In section 2 of this report are given the underlying ideas of this approach, summary of the difference equations and a discussion of the solution technique. Based on this approach a computer code, called NEMCO (numerical modeling of confined flows), has been developed. This code computes basically two-dimensional mass, momentum, and energy transfer. Thus it can handle flow which is either strictly two-dimensional--flow through nozzles, pipes, conical diffusers, discs, etc.--or is a two-dimensional approximation to the actual flow. An example of the latter is flow through a rectangular channel where the calculations are carried out along one pair of the opposing walls and the presence of the other pair is either completely ignored (case of large aspect ratio) or approximately taken into account by introducing some sort of effective width for the actual width between the ignored walls.

In section 3 is given the program listing of the code NEMCO-U2-79 in FORTRAN IV language. The program listing given was derived, with some FORTRAN language modifications, from the various versions of this code we have used in our computations. To check the code listing included in this report we have run two sets of sample calculations--one for flow through a pipe and another for flow through a rectangular diffuser. The results of the sample calculations are also given in section 3.

2. STREAMWISE COMPUTATION OF DUCT FLOWS

In this approach finite difference equations are obtained by first partitioning the flow in the duct into a finite number of streams and then applying conservation principles directly to the individual streams. To partition the flow into streams we draw a series of streamlines $1,2,\ldots,N$ as shown in Fig. 2. The direction x is along the flow and z, measured from the duct wall, normal to it. Duct wall is chosen as streamline 1 and the centerline or the symmetry plane as streamline N. We define streamline $j+1/2$ as the streamline in the middle of j and $j+1$. Streamline $j-1/2$ is defined similarly. We now define the various streams as follows:

Stream 2; flow between the wall and the streamline $2+1/2$,

Stream j $(N > j > 2)$; flow between streamlines $j-1/2$ and $j+1/2$,

Stream N; flow between the streamline $N-1/2$ and the centerline or the symmetry plane.

Shape of a typical stream depends on the shape of the duct: in rectangular ducts the streams will be plane layers and in circular ducts the streams will be cylindrical shells.

We introduce the following nomenclature:

ψ_j^+; mass flow rate between the streamline j and $j+1/2$.

ψ_j^-; mass flow rate between the streamline j and $j-1/2$.

$u_j(x)$, $T_j(x)$, $\rho_j(x)$, $h_j(x)$, $C_j(x)$; velocity, temperature, density, enthalpy and constant pressure specific heat along the streamline j.

$z_j(x)$; distance of streamline j from the wall.

A_j^X; cross-sectional area of stream j.

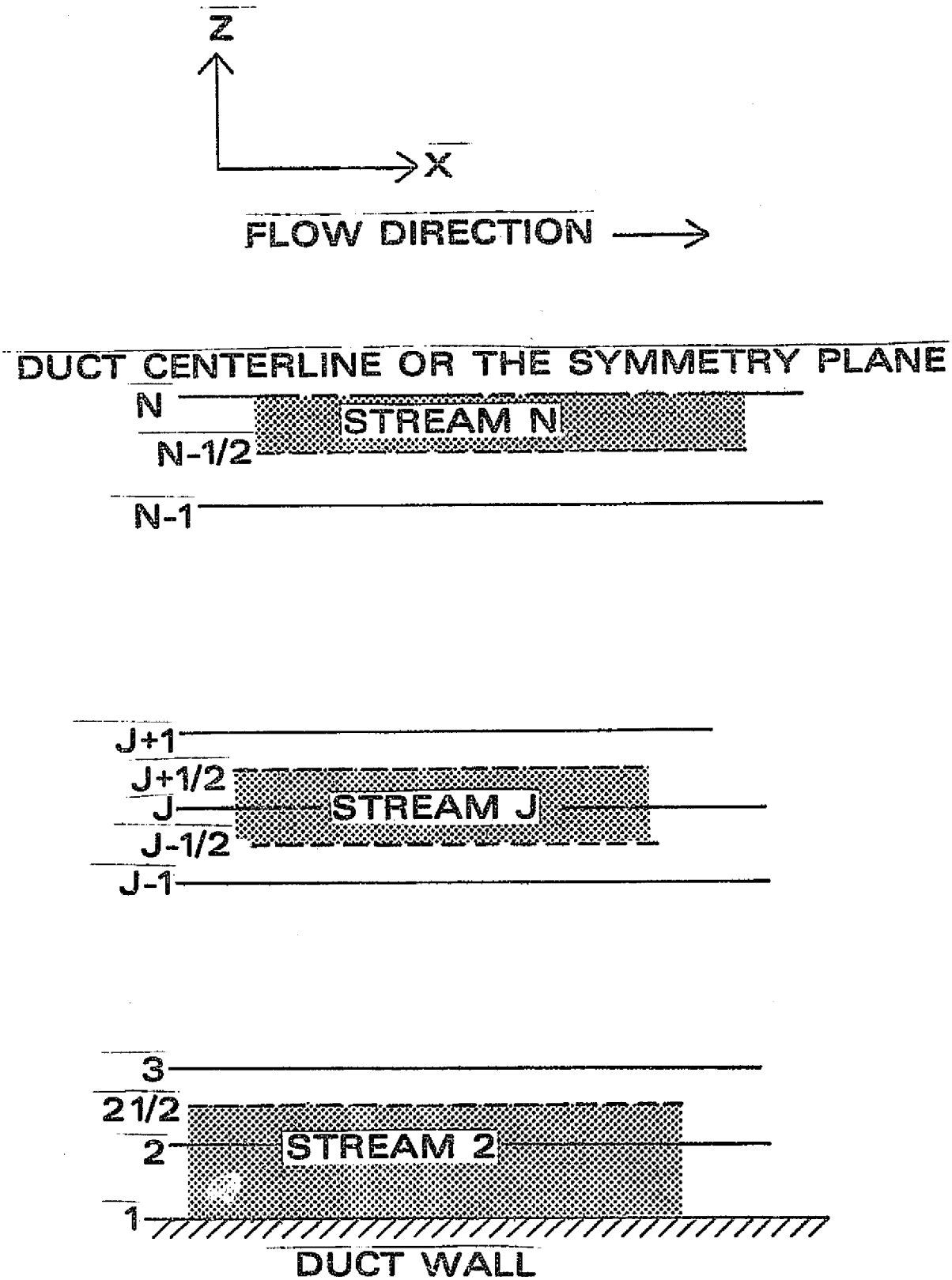
Z
X
FLOW DIRECTION
DUCT CENTERLINE OR THE SYMMETRY PLANE
N
N-1/2
STREAM N
N-1
J+1
J+1/2
J
J-1/2
J-1
STREAM J
3
21/2
2
1
STREAM 2
DUCT WALL

FIG. 2

$A^s_{j+1/2}$; interface area between streams j and j + 1.

$\mu_{j+1/2}(x)$, $k_{j+1/2}(x)$; effective viscosity and thermal conductivity (including turbulent contribution) along streamline j + 1/2.

Δp; pressure difference between x and x + Δx.

$V_{j+1/2} = \frac{\mu_{j+1/2}}{z_{j+1} - z_j} A^s_{j+1/2}$, $H_{j+1/2} = \frac{V_{j+1/2}}{Pr_{j+1/2}}$ where P_r is Prandtl's number.

α, β, δ; correction factors.

Applying conservation principles to individual streams across x and x + Δx we obtain the following finite difference equations for velocities and enthalpies (see Ref. 7):

Stream 2

$$-u_3^+\left[-\frac{\psi_2^+}{4} + \frac{1}{2}V^+_{2+1/2}\right] + u_2^+\left[\frac{3}{4}\psi_2^+ + \alpha\psi_2^- + \frac{1}{2}V^+_{2+1/2} + \frac{1}{2}V_1^+\right]$$

$$= -\Delta p A_2^x + u_3\left[\frac{\psi_2^+}{4} + \frac{1}{2}V_{2+1/2}\right] + u_2\left[\frac{3}{4}\psi_2^+ + \alpha\psi_2^- - \frac{1}{2}V_{2+1/2} - \frac{1}{2}V_1\right]. \quad (2\text{-}1)$$

$$- h_3^+\left(-\frac{\psi_2^+}{4} + \frac{1}{2}H^+_{2+1/2}\right) + h_2^+\left(\frac{3}{4}\psi_2^+ + \beta\psi_2^- + \frac{1}{2}H^+_{2+1/2} + \frac{1}{2}H_1^+\right)$$

$$= h_3\left(\frac{\psi_2^+}{4} + \frac{1}{2}H_{2+1/2}\right) + h_2\left(\frac{3}{4}\psi_2^+ + \beta\psi_2^- - \frac{1}{2}H_{2+1/2} - \frac{1}{2}H_1\right)$$

$$+ \frac{{u_3^+}^2}{2}\left(\frac{\psi_2^+}{4} + \frac{1}{2}V^+_{2+1/2}\right) - \frac{{u_2^+}^2}{2}\left(\frac{3}{4}\psi_2^+ + \delta\psi_2^- + \frac{1}{2}V^+_{2+1/2}\right)$$

$$+ \frac{u_3^2}{2}\left(\frac{\psi_2^+}{4} + \frac{1}{2}V_{2+1/2}\right) + \frac{u_2^2}{2}\left(\frac{3}{4}\psi_2^+ + \delta\psi_2^- - \frac{1}{2}V_{2+1/2}\right) + h_w\frac{1}{2}H_1^+ + h_w\frac{1}{2}H_1. \quad (2\text{-}2)$$

In Eq. (2-2) h_w represents enthalpy evlaluted at the wall temperature. This program assumes that the wall temperature is given. When wall heat flux is specified see Ref. 7 for the needed modification.

Streams ~~j~~ = 3,4,...,N-1

$$-u^+_{j+1}\left[-\frac{\psi_j^+}{4}+\frac{1}{2}V^+_{j+1/2}\right]+u_j{}^+\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- + \frac{1}{2}V^+_{j+1/2}+\frac{1}{2}V^+_{j-1/2}\right]$$

$$-u^+_{j-1}\left[-\frac{\psi_j^-}{4}+\frac{1}{2}V^+_{j-1/2}\right]$$

$$=-\Delta pA_j{}^x + u_{j+1}\left[\frac{\psi_j^+}{4}+\frac{1}{2}V_{j+1/2}\right]+u_j\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- - \frac{1}{2}V_{j+1/2}-\frac{1}{2}V_{j-1/2}\right]$$

$$+u_{j-1}\left[\frac{\psi_j^-}{4}+\frac{1}{2}V_{j-1/2}\right]. \qquad (2\text{-}3)$$

$$-h^+_{j+1}\left[-\frac{\psi_j^+}{4}+\frac{1}{2}H^+_{j+1/2}\right]+h_j{}^+\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- + \frac{1}{2}H^+_{j+1/2}+\frac{1}{2}H^+_{j-1/2}\right]$$

$$-h^+_{j-1}\left[-\frac{\psi_j^-}{4}+\frac{1}{2}H^+_{j-1/2}\right]$$

$$=h_{j+1}\left[\frac{\psi_j^+}{4}+\frac{1}{2}H_{j+1/2}\right]+h_j\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- - \frac{1}{2}H_{j+1/2}-\frac{1}{2}H_{j-1/2}\right]$$

$$+h_{j-1}\left[\frac{\psi_j^-}{4}+\frac{1}{2}H_{j-1/2}\right]+\frac{u^{+2}_{j+1}}{2}\left[-\frac{\psi_j^+}{4}+V^+_{j+1/2}\right]$$

$$-\frac{u_j{}^{+2}}{2}\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- + V^+_{j+1/2}+V^-_{j-1/2}\right]+\frac{u^{+2}_{j-1}}{2}\left[-\frac{\psi_j^-}{4}+V^+_{j-1/2}\right]$$

$$+\frac{u^2_{j+1}}{2}\left[\frac{\psi_j^+}{4}+V_{j+1/2}\right]+\frac{u_j{}^2}{2}\left[\frac{3}{4}\psi_j{}^+ + \frac{3}{4}\psi_j{}^- - V_{j+1/2}-V_{j-1/2}\right]$$

$$+\frac{u^2_{j-1}}{2}\left[\frac{\psi_j^-}{4}+V_{j-1/2}\right]. \qquad (2\text{-}4)$$

Stream N

Difference equations for stream N are obtained from Eqs. (2-3) and (2-4) by letting $j = N$, and setting

$$\psi_N^+ = V_{N+1/2}^+ = V_{N+1/2} = u_{N+1}^+ = u_{N+1} = 0, \quad \text{and}$$

$$h_{N+1}^+ = h_{N+1} = H_{N+1/2}^+ = H_{N+1/2} = 0$$

In the equations superscripts + indicates the quantities at $x + \Delta x$, thus u_j is the velocity along j at x and u_j^+ along j at $x + \Delta x$. In the listing given in section 3 same symbol is used for a given variable, like u_j, at x and $x + \Delta x$. Program is written such that from the point calculations of new quantities are started there is no further need for old values at x. Equations (2-1) to (2-4) are supplemented with formulae given in Appendices A and B of Ref. 7. These formulae calculate A^X, A^S, μ, and first at x calculate ψ_j^+ and ψ_j^- given ρ_j, u_j, and z_j and then later at $x + \Delta x$ calculate z_j^+ from ψ_j^+, ψ_j^-, ρ_j^+, and u_j^+.

The technique to solve these difference equations depends on the specified design variables. The program given in section 3 assumes that the variation of duct area along the flow is specified and the pressure gradient is unknown. For this case the difference equations are solved as follows. Equations (2-1) and (2-3) (and similarly Eqs. (2-2) and (2-4)) are looked at as a system of tri-diagonal linear algebraic equations. This, of course, is true only if V_j^+ and Δp are known. To start with we set V_j^+ equal to V_j and solve for u_j^+ using standard methods to solve tri-diagonal equations. We then calculate V_j^+ from the new u_j^+ and solve

for u_j^+ again using the new V_j^+. This iteration process is continued until new u_j^+ differs from the previously calculated u_j^+ by less than some preassigned value. In the early computations u_2, among all u_j, was found to change most rapidly from one iteration to another. Thus it has been chosen for all our computations to check for convergence. This iteration loop is called the velocity iteration loop. Solution for h_j^+ is carried on along with the velocity iteration calculating H_j^+ from the latest V_j^+ and using the latest available u_j^+ for kinetic energy terms.

The pressure drop Δp is also not known in advance. We start the calculations with some estimated Δp and carry out the velocity iteration. When the velocity (u_2^+) convergence is achieved we check the calculated duct area at $x + \Delta x$ against the specified duct area. If the difference is larger than some preassigned value we go back, adjust Δp, and start the velocity iteration all over agian. Iteration on Δp is called pressure iteration. Note the pressure iteration loop is placed over and above the velocity iteration loop. Initial estimate of Δp at the beginning of a new integration step is obtained by projecting the previous three values using least square method as discussed in Appendix A. Correction to Δp from one pressure iteration to the next is based on the difference between the calculated and the specified duct area.

3. PROGRAM LISTING

3.1 Introduction.

The program consists of a MAIN PROGRAM and nine subroutines;

INPUT1 (Provides geometric data for the duct),

START (Sets up grid normal to the flow and assigns starting velocity, temperature, density and enthalpy profiles),

SIPSIM (Calculates ψ_j^+ and ψ_j^-, the mass flow rates through upper and lower parts of streams),

ZDZ (Calculates distance of streamlines from the wall),

XSAREA (Calculates cross-sectional and surface areas of the streams),

VSCSTY (Calculates laminar and turbulent viscosities),

TDSOLV (Solves tri-diagonal set of algebraic equations),

THICK (Calculates various thicknesses associated with the boundary layer),

HPTRHO (Calculates temperature and density given pressure and enthalpy).

The main program and five of the subroutines are common to all duct shapes. The four subroutines INPUT1, SIPSIM, ZDZ, and XSAREA depend on the shape of the duct. Subsections 3.2 through 3.6 contain computer listings in FORTRAN IV as follows:

Subsection 3.2; Main program and subroutines common to all duct shapes,

Subsection 3.3; The four subroutines for circular ducts,

Subsection 3.4; Sample computations--pipe flow,

Subsection 3.5; The four subroutines for rectangular ducts,

Subsection 3.6; Sample computations--rectangular diffuser flow. All computer listings are proceeded by definitions of the FORTRAN names used in the program.

DATA INPUT:

Data input to program is through MAIN program and subroutine INPUT1. Input statements have been placed at the very beginning of the programs. Data given through the various programs is as follows:

Main Program:

a) UCNTR, ZBL, TCNTR, TWALL, P, CP, GASK, GASR, PRNDL, DPDX--dynamic and thermodynamic flow data.

b) XEND--length of the duct.

c) DPDTL, DXFAC, NJ, MAXIT, NPRNT--pressure gradient convergence tolerance, integration step size, number of grid points normal to the flow, maximum number of iterations allowed, and output frequency.

d) VZERO, TZERO, EXPV--parameters needed in subroutine VSCSTY to calculate laminar viscosity.

e) EXPU, EXPH--parameters needed in subroutine START to calculate initial boundary layer profiles.

INPUT1 Subroutine:

Duct shape, parameters differ for different duct shapes.

PROGRAM OUTPUT

Output of program is of three kinds:

a) Distribution of velocity, temperature, density, and enthalpy profiles normal to the flow at a given value of x. Frequency of this output is governed by NPRNT.

b) Distribution of pressure, shear stress, heat transfer to the wall, and various boundary layer parameters along x. These quantities are calculated at the end of each integration step and stored in vector forms. The results are printed after the calculations reach the end of the duct.

c) Many subroutines have output statements to print results of intermediate calculations. Frequency of this output is governed through the subscript appended to the subroutine name. This kind of output is used for diagnostic purposes.

3.2 Listing main program and common subroutines.

MAIN Program:

U(J) = Velocity along streamline J
T(J) = Temperature along streamline J
H(J) = Enthalpy along streamline J
RHO(J) = Density along streamline J
USH(J) = $(1/2)U^2(J)$
Z(J) = Distance from the wall of streamline J
DZ(J) = Z(J+1) - Z(J)
NJ = Number of grid points normal to the flow
NJ1 = NJ-1
NJ2 = NJ-2
NJP = NJ+1
NBL = Number of grid points assigned to the boundary layer at the inlet (NBL≤NJ)
Note: In this program NBL is calculated internally in subroutine START
DX = Size of integration step along the flow
SIP(J) = $\psi_j^+/4$
SIM(J) = $\psi_j^-/4$
FLORAT = Total mass flow rate through the duct
UCNTR = Velocity at the centerline or the symmetry plane
TCNTR = Temperature at the centerline or the symmetry plane
TWALL = Wall temperature
P = Pressure
CPINV = Inverse of C_p, the constant pressure specific heat
RINV = Inverse of R, the gas constant
GASK = Ratio of the specific heats
ZBL = Boundary layer thickness at the inlet
AX(J) = Cross-sectional area of stream J
AS(J) = Surface area between streams J and J-1

ALPHA, BETA, DELTA, GAMMA are the correction factors $\alpha,\beta,\delta,\gamma$ of Ref. 7.

RE = Reynolds number along streamline 2
VISC(J) = Viscosity along streamline J - 1/2
DEL1 = Displacement thickness of the boundary layer, δ_1
DEL2 = Momentum thickness of the boundary layer, δ_2
H12 = Shape factor DEL1/DEL2
DELO = Boundary layer thickness

COMMON/COM8; this common statement contains variable names pertaining to the duct shape. These names are defined later in INPUT1.

VF(J),HT(J) defined later by the program listing.

UU(J) = Intermediate velocity variable

The tri-diagonal form of the momentum and energy difference equation is

$$-a_j X_{j+1} + b_j X_j - c_j X_{j-1} = d_j \ ,$$

AA(J),BB(J),CC(J) are used for a_j, b_j, and c_j, first for momentum and then for energy equation; DD(J) is d_j for momentum equation and TT(J) d_j for the energy equation; DU(J) is part of DD(J) and DT(J) part of TT(S).

Note: Subscript I, in what follows, denotes the integration step along x.

EXVEC(I) = Distance along the flow, "x"
PVEC(I) = Pressure along x,
SHRVEC(I) = Shear stress at wall along x,
HTVEC(I) = Heat flux along x,
DLOVEC(I) = Boundary layer thickness along x,
DL1VEC(I) = Displacement thickness along x,
DL2VEC(I) = Momentum thickness along x,
SFVEC(I) = Shape factor along x.
CP = Constant pressure specific heat
GASR = Gas constant, R
PRNDL = Prandtle number
DPDX = Pressure gradient guess at the entrance

Note: The program requires as input a value of pressure gradient at the duct entrance for its first iteration. We suggest a value calculated approximately for one-dimensional flow.

XEND = Length of the duct
DPDTL = Tolerance within which dp/dx is to be calculated
DXFAC = Specifies the size of integration step, DX = HEIGHT/DXFAC
MAXIT = Maximum number of iterations allowed, if this number is exceeded the program will stop and write a message
NPRNT = Governs how often along x U(J),T(J), etc., are printed
VZERO,TZERO,EXPV: See subroutine VSCSTY
EXPU,EXPH; See subroutine START
XAMP,DXF,DX; for the first integration step DX is set equal to DXF/256, where DXF is calculated from DXF=HEIGHT/DXFAC. DX is multiplied by XAMP at the end of each integration step when DX reaches DXF XAMP is set equal to 1.
ISTEP = Integration step counter
ITRN = Iteration step counter
NPI = Counter for NPRNT
SNDSPD = Sound speed at the inlet
PTINL = Total pressure at the inlet
OLDA = Cross-sectional area of the duct at the previous integration step
DPDAMP, ZTLRN, UTLRN; See Appendix A

Symbols appearing in the rest of the MAIN program are either already defined or defined by FORTRAN statements appearing in this part of the program.

START Subroutine:

This subroutine sets up grid normal to the flow and specifies velocity, temperature, and density profiles at the beginning of the computations. The listing given assumes that the flow at the inlet has a boundary layer of thickness ZBL and that within the boundary layer velocity and enthalpy vary as

$$\frac{U(J)}{UCNTR} = \left(\frac{Z(J)}{ZBL}\right)^{1/EXPU} \qquad \text{and}$$

$$\frac{H(J) - HWALL}{HCNTR - HWALL} = \left(\frac{Z(J)}{ZBL}\right)^{1/EXPH}$$

where HWALL and HCNTR refer to enthalpies calculated at wall and centerline temperatures, and EXPU and EXPH are input parameters. Outside the boundary layer H and U are assumed to be uniform.

To set up grid that will compute the flow with maximum possible accuracy is difficult. Algorithm given in this subroutine is guided by the following considerations:

1. Grid points are to have greater density near the wall. The grid spacing to increase as Z to the some power, where Z is the distance from wall.
2. Grid point Z(2) (Z(1) is wall) is to be located just outside the laminar sublayer and the transition zone--the formula used to calculate turbulent viscosity for points Z(J), $J \geq 2$, is good only for the turbulent region. Also point Z(2) must not be too far away from the wall or we will lose accuracy in calculating flow near the wall.

3. Grid point Z(3) is to be selected so that Z(3)-Z(2) is not less than Z(2)-Z(1). Otherwise, as noted in Ref. 8, for some flow conditions one may encounter numerical instability.

In the subroutine Reynold's number along Z(2) is required to be around 1200. Thus U(2) and Z(2) are calculated from the two simultaneous equations,

$$\frac{U(2)}{UCNTR} = \left(\frac{Z(2)}{ZBL}\right)^{1./EXPU} \quad \text{and}$$

$$U(2)\ Z(2)/VISKIN = 1200$$

where VISKIN is kinematic viscosity.

Grid location of Z(J), J≥2, are next calculated using a variation of formula given in Ref. 8. We put

$$Z(J) = Z(2) + (HEIGHT - Z(2))\left(\frac{J}{NJ}\right)^{EXPG}$$

Grid exponent EXPG is calculated from the requirement

$$Z(3) - Z(2) = Z(2) - Z(1)$$

VSCSTY Subroutine:

This subroutine calculates viscosity (laminar + turbulent) coefficients along the stream interfaces. Symbol VISC(J) represents the viscosity coefficient along streamline J - 1/2. The laminar contribution to VISC(J) is calculated from the formula

$$VISC(J) = \frac{1}{2}\left[VZERO\left\{\frac{T(J)}{TZERO}\right\}^{EXPV} + VZERO\left\{\frac{T(J-1)}{TZERO}\right\}^{EXPV}\right]$$

where VZERO, EXPV, and TZERO are constants and are required input to the subroutine. The turbulent contribution to VISC(J) is calculated from

$$VISC(J) = \left[\frac{EL(J) + EL(J-1)}{2}\right]^2 \times \left[\frac{RHO(J) + RHO(J-1)}{2}\right] \times \left|\frac{U(J) - U(J-1)}{DZ(J)}\right|$$

The mixing length, EL, is calculated from the relation for the fully developed turbulent pipe flow [Eq. (20.18), Ref. 1].

$$EL(J) = ZOUT\left[0.14 - 0.08\left(\frac{1 - Z(J)}{ZOUT}\right) - 0.06\left(\frac{1 - Z(J)}{ZOUT}\right)^2\right]$$

The quantity 1 - Z(J)/ZOUT represents the distance from the centerline normalized by half the channel width.

This subroutine also calculates the correction factors γ, α, δ, and β. The formulae used in the subroutine to calculate γ, α, and δ were obtained by curve fitting the graphs shown in Ref. 7. Factor β

was calculated from α using the relation given in Ref. 8. (This relation is obtained using integrals defining α and β, and assuming $\frac{h - h_w}{h - h_0} \sim \frac{u}{u_0}$).

The formula to calculate FF from Re was obtained by curve fitting the relation shown in Fig. B-1 of Ref. 7 in the vicinity of RE ~ 1000.

TDSOLV Subroutine:

This subroutine solves the tri-diagonal set of equations

$$-A_j X_{j-1} + B_j X_j - C_j X_{j+1} = D_j$$

where $j = 2,3,\ldots,N$. Method used is the well-known special case of Gauss' reduction formula (see for example Ref. 2). The boundary condition at the wall is already built into the difference equations and appears as

$$C_2 = 0.$$

The boundary condition at the center line used in the present work is that of symmetry and appears as

$$A_N = 0.$$

THICK Subroutine:

This subroutine calculates boundary layer thickness, δ_0, displacement thickness, δ_1, momentum thickness, δ_2, and shape factor H_{12}.

Boundary layer thickness is defined as the distance from the wall where the velocity equals 0.995 times center line velocity. The integrals

$$\delta_1 = \int_0^\infty \left[1 - \frac{\rho u}{\rho_\infty u_\infty}\right] dZ , \qquad \text{and}$$

$$\delta_2 = \int_0^\infty \frac{\rho u}{\rho_\infty u_\infty}\left[1 - \frac{\sigma u}{\sigma_\infty u_\infty}\right] dZ$$

were approximated by the sums

$$\delta_1 = \left(1 - \frac{\gamma \rho_2 u_2}{\rho_\infty u_\infty}\right) dZ_2 + \sum_{j=3}^{N} \frac{1}{2}\left[\left(1 - \frac{\rho_j u_j}{\rho_\infty u_\infty}\right) + \left(1 - \frac{\rho_{j-1} u_{j-1}}{\rho_\infty u_\infty}\right)\right] dZ_j$$

and

$$\delta_2 = \left(\frac{\gamma \rho_2 u_2}{\rho_\infty u_\infty} - \frac{\alpha\gamma \rho_2 u_2^2}{\rho_\infty u_\infty^2}\right) dZ_2 + \sum_{j=3}^{N} \frac{1}{2}\left[\frac{\rho_j u_j}{\rho_\infty u_\infty}\left(1 - \frac{u_j}{u_\infty}\right) + \frac{\rho_{j-1} u_{j-1}}{\rho_\infty u_\infty}\left(1 - \frac{u_{j-1}}{u_\infty}\right)\right] dZ_j$$

HPTRHO Subroutine:

This subroutine calculates temperature and density given pressure and enthalpy. In its present form it uses formulae for perfect gas with constant specific heats.

```
C
C*****NEMCO-U2-79*************************************************************
C
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM3/SIP(50),SIM(50),FLORAT
      COMMON/COM4/UCNTR,TCNTR,TWALL,P,CPINV,RINV,GASK,ZBL
      COMMON/COM5/AX(50),AS(50)
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM7/DEL1,DEL2,H12,DELO
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
      COMMON/COM9/VZERO,EXPV,TZERO,EXPU,EXPH
      DIMENSION VF(50),HT(50),UU(50)
      DIMENSION AA(50),BB(50),CC(50),DD(50),DU(50),DT(50),TT(50)
      DIMENSION EXVEC(200),PVEC(200),PRVEC(200),SHRVEC(200),
     1HTVEC(200),DLOVEC(200),DL1VEC(200),DL2VEC(200),SFVEC(200)
C
C**DATA INPUT ****************************************************************
C**DYNAMIC AND THERMODYNAMIC DATA*************************
      UCNTR=[redacted]
      ZBL=[redacted]
      TCNTR=[redacted]
      TWALL=[redacted]
      P=[redacted]
      CP=[redacted]
      GASK=[redacted]
      GASR=[redacted]
      PRNDL=[redacted]
      DPDX=[redacted]
C**GEOMETRIC DUCT DATA*******************************************
      CALL INPUT1
      XEND=[redacted]
C**CONVERGENCE CRITERIA,GRID SIZE,AND OTHER DATA=**********
      DPDTL=[redacted]
      DXFAC=[redacted]
      MAXIT=[redacted]
      NJ=[redacted]
      NPRNT=[redacted]
C**DATA FOR VSCSTY AND START SUBRCUTINES*********************
      VZERO=[redacted]
      TZERO=[redacted]
      EXPV=[redacted]
      EXPU=[redacted]
      EXPH=[redacted]
C
C**INITIALIZATIONS*************************************************************
      XAMP=2.0
      ISTEP=0
      ITRN=0
      NPI=NPRNT-1
      NJP=NJ+1
      NJ1=NJ-1
      NJ2=NJ-2
      SIP(NJ)=0.0
```

```
      VF(NJP)=0.
      HT(NJP)=0.
      U(NJP)=0.
      H(NJP)=0.
      USH(NJP)=0.
      CC(2)=0.
      DXF=HEIGHT/DXFAC
      DX=DXF/256.
      EX=-DX
      DP=DPDX*DX
      PRLINV=1./PRNDL
      CPINV=1./CP
      RINV=1./GASR
      GASKIN=1./GASK
      HWALL=CP*TWALL
      SNDSPD=(GASK*GASR*TCNTR)**0.5
      PTINL=P*(1.+.5*(GASK-1.)*(UCNTR/SNDSPD)**2)**(GASK/(GASK-1.))
      PINL=P
      DPPTI=PTINL-PINL
      DP1=0.
      DP2=0.
      DX1=0.
      OLDA=0.0
      CALL START
      WRITE(6,30)
   30 FORMAT(1H1,' VELOCITY U,TEMPERATURE T,DENSITY RHO, AND ENTHAPY H
     1DISTRIBUTIONS AS  FUNCTION OF Z.',//)
C
C**AXIAL CALCULATIONS*************************************************
 1100 ISTEP=ISTEP+1
      EX=EX+DX
      CALL VSCSTY(0)
      CALL THICK
      NPI=NPI+1
      EXVEC(ISTEP)=EX
      PVEC(ISTEP)=P
      PRVEC(ISTEP)=(P-PINL)/DPPTI
      SHRVEC(ISTEP)=VISC(2)*U(2)
      HTVEC(ISTEP)=VISC(2)*PRLINV*(H(2)-HWALL)
      DLOVEC(ISTEP)=DEL0
      DL1VEC(ISTEP)=DEL1
      DL2VEC(ISTEP)=DEL2
      SFVEC(ISTEP)=H12
      IF(NPI.LT.NPRNT) GO TO 1200
      NPI=0
      WRITE(6,46)EX,RE
   46 FORMAT('***DIST. ALONG X=',F7.4,'****NEAR WALL RE=',1PE10.4)
      WRITE(6,40) (Z(J),J=2,NJ)
      WRITE(6,41) (U(J),J=2,NJ)
      WRITE(6,42) (T(J),J=2,NJ)
      WRITE(6,43) (RHO(J),J=2,NJ)
      WRITE(6,44) (H(J),J=2,NJ)
   40 FORMAT(/,1X,'Z',8X  ,1P10E11.4/(10X,1P10E11.4))
   41 FORMAT(/,1X,'U',8X  ,1P10E11.4/(10X,1P10E11.4))
```

```
   42 FORMAT(/,1X,'T',8X  ,1P10E11.4/(10X,1P10E11.4))
   43 FORMAT(/,' RHO',6X  ,1P10E11.4/(10X,1P10E11.4))
   44 FORMAT(/,1X,'H',8X  ,1P10E11.4/(10X,1P10E11.4))
      IF (EX.GT.XEND) GO TO 2100
C
C** RESETTING DYNAMIC,THERMO, AND GEOM. VARIABLES FOR NEXT ITERATION*****
 1200 ITRN=1
      X1=DX1
      X2=DX+DX1
      SUMX=X1+X2
      SUMXX=X1*X1+X2*X2
      SUMP=DP+DP1+DP2
      SUMXP=X1*DP1+X2*DP
      DEN=3.*SUMXX-SUMX*SUMX
      DPSLOP=(3.*SUMXP-SUMP*SUMX)/DEN
      DPZERO=(SUMP*SUMXX-SUMX*SUMXP)/DEN
      DX1=DX
      DP2=DP1
      DP1=DP
      IF(ISTEP.LE.2) GO TO 1205
      IF(DX.GE.DXF) XAMP=1.0
      DX=DX*XAMP
      IF(ISTEP.LT.4) GO TO 1205
      DP=DPSLOP*(X2+DX)+DPZERO
 1205 PSTART=P
      DPDAMP=0.5*EXP(-0.115*EX/HEIGHT)
      UOLD=U(2)
      CALL SIPSIM(0)
      CALL XSAREA(0)
      ZFAC=FLORAT*U(NJ)/(OLDA-FLORAT*U(NJ)*GASKIN/P)
      ADP=ABS(DP1)
      ZTLRN=ADP*DPDTL*HEIGHT/ZFAC/ARFAC
      UTLRN=ADP*DPDTL*OLDA/FLORAT
C
C**** EVALUATION OF R.H.S. OF MOM. AND ENERGY EQS. ********************
      VF(2)=.5*AS(2)*VISC(2)
      HT(2)=VF(2)*PRLINV
      DO 1106 J=3,NJ
      VF(J)=.5*AS(J)*VISC(J)/DZ(J)
 1106 HT(J)=VF(J)*PRLINV
      DU(2)=U(3)*(SIP(2)+VF(3))+U(2)*(3.*SIP(2)+ALPHA*SIM(2)-VF(3)-VF(2)
     1)
      DD(2)=DU(2)-DP*AX(2)
      DO 1108 J=3,NJ
      DU(J)=U(J+1)*(SIP(J)+VF(J+1))+U(J)*(3.*SIP(J)+3.*SIM(J)-VF(J+1)
     1-VF(J))+U(J-1)*(SIM(J)+VF(J))
      DD(J)=DU(J)-DP*AX(J)
 1108 CONTINUE
      DT(2)=H(3)*(SIP(2)+HT(3))+H(2)*(3.*SIP(2)+BETA*SIM(2)-HT(3)-HT(2))
     1+USH(3)*(SIP(2)+VF(3))+USH(2)*(3.*SIP(2)+DELTA*SIM(2)-VF(3))+HWALL
     2*HT(2)
      DO 1112 J=3,NJ
      DT(J)=H(J+1)*(SIP(J)+HT(J+1))+H(J)*(3.*SIP(J)+3.*SIM(J)-HT(J+1)
     1-HT(J))+H(J-1)*(SIM(J)+HT(J))+USH(J+1)*(SIP(J)+VF(J+1))+USH(J)*(
```

```
     23.*SIP(J)+3.*SIM(J)-VF(J+1)-VF(J))+USH(J-1)*(SIM(J)+VF(J))
 1112 CONTINUE
C
C**ITERATIVE CALCULATIONS FOR NEW U ,H ,Z ********************************
C**CALCULATIONS OF NEW VELOCITIES************************
      GO TO 95
   90 CONTINUE
      VF(2)=AS(2)*VISC(2)*0.5
      HT(2)=VF(2)*PRLINV
      DO 99 J=3,NJ
      VF(J)=AS(J)*VISC(J)/DZ(J)*0.5
   99 HT(J)=VF(J)*PRLINV
   95 AA(2)=-SIP(2)+VF(3)
      BB(2)=3.*SIP(2)+ALPHA*SIM(2)+VF(3)+VF(2)
      DO 100 J=3,NJ
      AA(J)=-SIP(J)+VF(J+1)
      BB(J)=3.*SIP(J)+3.*SIM(J)+VF(J+1)+VF(J)
      CC(J)=-SIM(J)+VF(J)
  100 CONTINUE
      CALL TDSOLV (AA,BB,CC,DD,UU,0)
      IF (ITRN.GT.1) GO TO 109
      DO 108 J=2,NJ
  108 U(J)=UU(J)
  109 DO 200 J=2,NJ
  200 U(J)=.5*(U(J)+UU(J))
C
C**CALCULATION OF NEW ENTHALPIES*****************************
      DO 111 J=2,NJ
  111 USH(J)=.5*U(J)*U(J)
      AA(2)=-SIP(2)+HT(3)
      BB(2)=3.*SIP(2)+BETA*SIM(2)+HT(3)+HT(2)
      TT(2)=DT(2)+USH(3)*(-SIP(2)+VF(3))-USH(2)*(3.*SIP(2)+DELTA*SIM(2)
     1+VF(3))+HWALL*HT(2)
      DO 115 J=3,NJ
      AA(J)=-SIP(J)+HT(J+1)
      BB(J)=3.*SIP(J)+3.*SIM(J)+HT(J+1)+HT(J)
      CC(J)=-SIM(J)+HT(J)
      TT(J)=DT(J)+USH(J+1)*(-SIP(J)+VF(J+1))-USH(J)*(3.*SIP(J)+3.*SIM(J)
     1+VF(J+1)+VF(J))+USH(J-1)*(-SIM(J)+VF(J))
  115 CONTINUE
      CALL TDSOLV(AA,BB,CC,TT,H,0)
C
      P=PSTART+DP
      CALL HPTRHO
      CALL ZDZ
C**CONVERGENCE CHECKS ***************************************************
      ZCOR=Z(NJ)-HEIGHT
   80 ITRN=ITRN+1
      UCHEK=ABS(U(2)-UOLD)
      ZCHEK=ABS(ZCOR)
      IF(UCHEK.GT.UTLRN)GO TO 82
      IF(ZCHEK.LT.ZTLRN)GO TO 1100
      IF (ITRN.GT.MAXIT) GO TO 2000
      DPCOR=-ZCOR*ARFAC/Z(NJ)*ZFAC
```

```
      310 DPCOR=DPCOR*DPDAMP
      330 DP=DP+DPCOR
          DO 340 J=2,NJ
      340 DD(J)=DU(J)-DP*AX(J)
       82 CONTINUE
          UOLD=U(2)
          CALL VSCSTY(0)
          GO TO 90
C
     2000 WRITE(6,2001)
     2001 FORMAT(/,1X,'**VELOCITY-PRESSURE ITERATIONS EXCEEDED THE LIMIT SET
         1 BY MAXIT**********')
     2100 WRITE(6,2101)
     2101 FORMAT(1H1,9X,'EX',9X,'PRESS',9X,'P-RECV',6X,'WALL SHEAR'
         1,3X,'HEAT FLX',5X,'B.L. THK',5X,'DISP THK',5X,'MOM THK',6X,'SHAPE
         2FAC',/)
          WRITE(6,2105) ( EXVEC(K),PVEC(K),PRVEC(K),SHRVEC(K),
         1HTVEC(K),DLOVEC(K),DL1VEC(K),DL2VEC(K),SFVEC(K), K=1,ISTEP)
     2105 FORMAT(1PE16.3,1P8E13.3)
          STOP
          END
```

```
      SUBROUTINE START
C**SUBROUTINE SETS INITIAL PROFILES AND NORMAL TO THE FLOW GRID*********
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM4/UCNTR,TCNTR,TWALL,P,CPINV,RINV,GASK,ZBL
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
      COMMON/COM9/VZERO,EXPV,TZERO,EXPU,EXPH
C
      VISKIN=VZERO*(TCNTR/TZERO)**EXPV/(P*RINV/TCNTR)
      FNJ=FLOAT(NJ)
      HCNTR=TCNTR/CPINV
      HWALL=TWALL/CPINV
      DELH=HCNTR-HWALL
      U2Z2=1200.*VISKIN
      U(1)=0.
      Z(1)=0.
      DZ(1)=0.
      H(1)=HWALL
      U(2)=(UCNTR**EXPU*U2Z2/ZBL)**(1./(1.+EXPU))
      Z(2)=U2Z2/U(2)
      DZ(2)=Z(2)-Z(1)
      H(2)=HWALL+DELH*(Z(2)/ZBL)**(1./EXPH)
      EXPG=ALOG(HEIGHT/Z(2)-1.)/ALOG(FNJ/3.)
      DO 20 J=3,NJ
      FJ=FLOAT(J)
      Z(J)=Z(2)+(HEIGHT-Z(2))*(FJ/FNJ)**EXPG
   20 DZ(J)=Z(J)-Z(J-1)
      DO 23 J=3,NJ
      U(J)=UCNTR*(Z(J)/ZBL)**(1./EXPU)
      H(J)=HWALL+DELH*(Z(J)/ZBL)**(1./EXPH)
      IF(U(J).GE.UCNTR) GO TO 25
   23 CONTINUE
      GO TO 27
   25 DO 26 K=J,NJ
      U(K)=UCNTR
   26 H(K)=HCNTR
   27 CONTINUE
      DZ(NJ+1)=0.
      PRINV=P*RINV
      DO 223 J=2,NJ
      T(J)=H(J)*CPINV
  223 RHO(J)=PRINV/T(J)
      T(1)=TWALL
      RHO(1)=P*RINV/T(1)
      DO 31 J=1,NJ
   31 USH(J)=U(J)*U(J)*0.5
      RETURN
      END
```

S FORTRAN IV 360N-FO-479 3-8 VSCSTY DATE 08/20/79 TIME 13.

```
      SUBROUTINE VSCSTY(NP)
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM9/VZERO,EXPV,TZERO,EXPU,EXPH
      DIMENSION EL(50)
C**MIXING LENGTH , EL, CALCULATIONS***************
      ZOUT=Z(NJ)
      EL(1)=0.
      DO 210 J=2,NJ
      FAC=(1.0-Z (J)/ZOUT)**2
      EL (J)=ZOUT*(0.14-0.08*FAC-0.06*FAC*FAC)
  210 CONTINUE
C**VISCOSITY (LAMINAR+TURBULENT) CALCULATIONS******
      VISC(1)=VZERO*(T(1)/TZERO)**EXPV
      VISOLD =VZERO*(T(2)/TZERO)**EXPV
      RE=U(2)*RHO(2)*Z(2)/VISOLD
      DO215 J=3,NJ
      VISNEW =VZERO*(T(J)/TZERO)**EXPV
      VISL=0.5*(VISOLD+VISNEW)
      VISOLD=VISNEW
      ELTS=((EL(J)+EL(J-1))*0.5)**2
      VIST=ELTS*(RHO(J)+RHO(J-1))*0.5
      DUJ=ABS(U(J)-U(J-1))
      VIST=VIST*DUJ/DZ(J)
      VISC (J)=VISL+VIST
  215 CONTINUE
      FF=0.0290106/RE**0.2827065
      VISC(2)=FF*RHO(2)*U(2)
      G=ALOG(RE)
      G2=G*G
      G3=G2*G
      GAMMA=0.6995594-0.1735247*G+0.04343642*G2-0.002524061*G3
      ALPHA=0.8311825-0.2551132*G+0.05791319*G2-0.003226784*G3
      DELTA=0.7987116-0.1194877*G+0.03075327*G2-0.001806241*G3
      BETA=(T(1)+ALPHA*(T(2)-T(1)))/T(2)
      IF (NP.NE.1) GO TO 999
      WRITE (6,102) (EL (J),J=1,NJ   )
      WRITE (6,103) (VISC (J),J=1,NJ   )
  102 FORMAT(/,' EL',7X,     10E11.3/(10X,10E11.3))
  103 FORMAT(/,' VISC',5X,   10E11.3/(10X,10E11.3))
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TDSOLV(A,B,C,D,X,NP)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      DIMENSION A(50),B(50),C(50),D(50),X(50),E(50),F(50)
C     *****  ROUTINE TO SOLVE THE TRIDIAGONAL EQUATIONS  **********
      E(2)=A(2)/B(2)
      F(2)=D(2)/B(2)
      DO 500 J=3,NJ1
      DEN=B(J)-C(J)*E(J-1)
      E(J)=A(J)/DEN
  500 F(J)=(D(J)+C(J)*F(J-1))/DEN
  505 X(NJ1)=(E(NJ1)*D(NJ)+B(NJ)*F(NJ1))/(B(NJ)-E(NJ1)*C(NJ))
      DO 510 I=2,NJ2
      J=NJ-I
  510 X(J)=E(J)*X(J+1)+F(J)
      X(NJ)=(D(NJ)+C(NJ)*X(NJ1))/B(NJ)
      IF (NP.NE.1) GO TO 999
      WRITE(6,102) (A(J),J=2,NJ1)
      WRITE(6,103) (B(J),J=2,NJ1)
      WRITE(6,104) (C(J),J=2,NJ1)
      WRITE(6,105) (D(J),J=2,NJ1)
      WRITE(6,106) (E(J),J=2,NJ1)
      WRITE(6,107) (F(J),J=2,NJ1)
  102 FORMAT(/,1X,'A',8X  ,1P10E11.4/(10X,1P10E11.4))
  103 FORMAT(/,1X,'B',8X  ,1P10E11.4/(10X,1P10E11.4))
  104 FORMAT(/,1X,'C',8X  ,1P10E11.4/(10X,1P10E11.4))
  105 FORMAT(/,1X,'D',8X  ,1P10E11.4/(10X,1P10E11.4))
  106 FORMAT(/,1X,'E',8X  ,1P10E11.4/(10X,1P10E11.4))
  107 FORMAT(/,1X,'F',8X  ,1P10E11.4/(10X,1P10E11.4))
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE THICK
C**** THIS SUBROUTINE CALCULATES DISP. AND MOM. THIKNESSES **********
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM7/DEL1,DEL2,H12,DELO
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
      DIMENSION RU1(50),RU2(50)
      UINV=1./U(NJ)
      RUINV=0.5/RHO(NJ)/U(NJ)
      DO 20 J=1,NJ
      RU1(J)=RHO(J)*U(J)*RUINV
   20 RU2(J)=RU1(J)*(1.-U(J)*UINV)
      DEL1=(1.-RU1(2)*2.*GAMMA)*DZ(2)
      DEL2=RU1(2)*2.*GAMMA*(1.-U(2)*ALPHA*UINV)*DZ(2)
      DO 30 J=3,NJ
      DEL1=DEL1+(1.-RU1(J)-RU1(J-1))*DZ(J)
   30 DEL2=DEL2+(RU2(J)+RU2(J-1))*DZ(J)
      H12=DEL1/DEL2
      UPBL=0.995*U(NJ)
      DO 40 J=2,NJ
      IF( U(J).GT.UPBL) GO TO 41
   40 CONTINUE
   41 DELO=Z(J-1)+DZ(J)*((UPBL-U(J-1))/(U(J)-U(J-1)))
      RETURN
      END
```

IS FORTRAN IV 360N-FO-479 3-8 HPTRHO DATE 08/20/79 TIME 13.

```
      SUBROUTINE HPTRHO
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM4/UCNTR,TCNTR,TWALL,P,CPINV,RINV,GASK,ZBL
      PRINV=P*RINV
      DO 223 J=2,NJ
      T(J)=H(J)*CPINV
  223 RHO(J)=PRINV/T(J)
      RETURN
      END
```

3.3 Listing of CIRCULAR Geometry Subroutines.

Most of the calculations in these subroutines are carried out in terms of the radial distance R(J) from the center line. Relations between R(J) and Z(J) is given by

$$R(J) = HEIGHT - Z(J)$$

INPUT1 Subroutine:

This subroutine provides data for the duct shape.

HEIGHT = Radius of the duct.
HSLOPE = Axial variation of the radius defined as:
HEIGHT at $x + \Delta x$ = HEIGHT at $x +$ HSLOPE$\cdot\Delta x$
ARFAC = 2. for circular ducts

SIPSIM and ZDZ Subroutines:

Given U(J), RHO(J), and Z(J), subroutine SIPSIM calculates SIP(J) and SIM(J). It is called at the beginning of each integration step. Subroutine ZDZ, called at the end of the integration step, calculates Z(J) given U(J), RHO(J), SIP(J), and SIM(J). Fortran statements in these routines are almost direct translations of the formulae given in Appendices A and B of Ref. 7.

XSAREA Subroutine:

This subroutine calculates cross-sectional and surface areas of the streams. It also calculates cross-sectional area of the duct called OLDA. This area is used in the pressure iteration formulae.

```
      SUBROUTINE INPUT1
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
      HEIGHT=[redacted]
      HSLOPE=[redacted]
      ARFAC=2.0
      RETURN
      END
```

```
      SUBROUTINE SIPSIM(NP)
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM3/SIP(50),SIM(50),FLORAT
      COMMON/COM5/AX(50),AS(50)
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM7/DEL1,DEL2,H12,DELO
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,CLDA,ARFAC
      DIMENSION R(50),RR(50),RSQ(50),RU(50)
      PI=3.141593
      PI16TH=PI/16.0
      FLORAT=0.0
      R(1)=HEIGHT
      DO 10 J=2,NJ
      RU(J)=RHO(J)*U(J)
      R(J)=HEIGHT-Z(J)
      RR(J)=R(J)*R(J)
   10 RSQ(J)=0.25*(R(J)+R(J-1))**2
C**SI'S, EXCEPT SIM(2), REDUCED BY 4 TO ABSORB 1/4 APPEARING IN SI(J)/4*
      DO 30 J=2,NJ1
      SIP(J)=PI16TH*(3.*RU(J)+RU(J+1))*(RR(J)-RSQ(J+1))
   30 FLORAT=FLORAT+ SIP(J)
      SIM(2)=RU(2)*GAMMA*(R(1)+R(2))*PI*(R(1)-R(2))
      FLORAT=FLORAT+ SIM(2)*0.25
      DO 40 J=3,NJ1
      SIM(J)=PI16TH*(3.*RU(J)+RU(J-1))*(RSQ(J)-RR(J))
   40 FLORAT=FLORAT+ SIM(J)
      SIM(NJ)=PI16TH*(3.*RU(NJ)+RU(NJ1))*RR(NJ1)*0.25
      FLORAT=4.0*FLORAT+4.0*SIM(NJ)
      IF(NP.NE.1) GO TO 100
      WRITE(6,43) ( SIM(J),J=3,NJ)
      WRITE(6,44) ( SIP(J),J=2,NJ1)
   43 FORMAT(/,' SIM',6X  ,10E11.3/(10X,10E11.3))
   44 FORMAT(/,' SIP',6X  ,10E11.3/(10X,10E11.3))
      WRITE(6,53) FLORAT
   53 FORMAT (/,10X,'FLOW RATE= ' , E11.4)
  100 RETURN
      END
```

```
      SUBROUTINE ZDZ
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM3/SIP(50),SIM(50),FLORAT
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
      DIMENSION R(50),RU(50)
      R(NJ)=0.
      PI=3.141593
      DO 10 J=2,NJ
   10 RU(J)=RHO(J)*U(J)
      R(NJ1)=(32.*(SIP(NJ1)+SIM(NJ1))/PI/(5.*RU(NJ1)+3.*RU(NJ)))**0.5
      DO 20 N=4,NJ
      J=NJ+3-N
      R(J-1)=(16.*SIP(J-1)/PI/(3.*RU(J-1)+RU(J))+16.*SIM(J)/PI/(3.*RU(J)
     1+RU(J-1))+R(J)*R(J))**0.5
   20 CONTINUE
      R(1)=(SIM(2)/(RU(2)*GAMMA*PI)+R(2)*R(2))**0.5
      DO 40 J=2,NJ
      Z(J)=R(1)-R(J)
   40 DZ(J)=Z(J)-Z(J-1)
      RETURN
      END
```

IS FORTRAN IV 360N-FO-479 3-8 XSAREA DATE 08/20/79 TIME 13.

```
      SUBROUTINE XSAREA(NP)
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM5/AX(50),AS(50)
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,CFWIDH,OLDA,APFAC
      DIMENSION R(50),RSQ(50)
      PI=3.141593
      PIDX=PI*DX
      R(1)=HEIGHT
      DO 10 J=2,NJ
      R(J)=HEIGHT-Z(J)
   10 RSQ(J)=0.25*(R(J)+R(J-1))**2
      AX(2)=PI* R(2)*(2.*R(1)-R(3)-R(2))
      AS(2)=PIDX*R(1)*2.
      DO 20 J=3,NJ1
      AX(J)=PI*(RSQ(J)-RS    ))
   20 AS(J)=PIDX*(R(J)+R(J-
      AX(NJ)=PI*RSQ(NJ)
      AS(NJ)=PIDX*(R(NJ)+R(NJ1))
      OLDA=PI*HEIGHT*HEIGHT
      HEIGHT=HEIGHT+HSLOPE*DX
      IF(NP.NE.1)GO TO 100
      WRITE (6,30) (AX(J),J=2,NJ)
   30 FORMAT (/,' AX',7X,10E11.3/(10X,10E11.3))
      WRITE(6,40) (AS(J),J=2,NJ)
   40 FORMAT(/,' AS',7X,10F11.3/(10X,10E11.3))
  100 RETURN
      END
```

3.4 Sample Computations--Entry Region Pipe Flow

DATA: (SI Units)

MAIN program:

```
C**DYNAMIC AND THERMODYNAMIC DATA*****************************
      UCNTR=15.0
      ZBL=0.01
      TCNTR=400.0
      TWALL=300.0
      P=70927.5
      CP=1003.5
      GASK=1.4
      GASR=287.0
      PRNDL=0.7
      DPDX=-5000.0
C**GEOMETRIC DUCT DATA***************************************
      CALL INPUT1
      XEND=10.0
C**CONVERGENCE CRITERIA,GRID SIZE,AND OTHER DATA*************
      DPDTL=0.01
      DXFAC=5.0
      MAXIT=50
      NJ=21
      NPRNT=10
C**DATA FOR VSCSTY AND START SUBROUTINES*********************
      VZERO=0.0000386
      TZERO=860.0
      EXPV=.7
      EXPU=7.
      EXPH=7.
C
```

INPUT1 Subroutine:

```
HEIGHT=0.5
HSLOPE=0.0
```

JOB STATISTICS:

Machine: IBM 360/44 with DOS operating system.
Cards Read: 500
Core Required: 44K
Compilation Time: 3.19 mns
CPU Time: 2.11 mns

**DIST. ALONG X= 0.0 ****NEAR WALL RE=1.2763E 03

Z	3.4106E-03	6.8213E-03	1.0533E-02	1.6020E-02	2.3519E-02	3.3246E-02	4.5403E-02	6.0179E-02	7.7751E-02	9.8291E-02
	1.2196E-01	1.4892E-01	1.7931E-01	2.1329E-01	2.5098E-01	2.9254E-01	3.3810E-01	3.8777E-01	4.4179E-01	5.0000E-01
U	1.2863E 01	1.4202E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01
	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01	1.5000E 01
T	3.8576E 02	3.9468E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02
	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02	4.0000E 02
RHO	6.4065E-01	6.2616E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01
	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01	6.1784E-01
H	3.8711E 05	3.9606E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05
	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05	4.0140E 05

DIST. ALONG X= 0.2000**NEAR WALL RE=1.2875E 03

Z	3.2813E-03	6.5789E-03	1.0195E-02	1.5497E-02	2.2629E-02	3.1838E-02	4.3335E-02	5.7283E-02	7.3833E-02	9.3117E-02
	1.1525E-01	1.4031E-01	1.6834E-01	1.9932E-01	2.3310E-01	2.6929E-01	3.0698E-01	3.4402E-01	3.7511E-01	5.0000E-01
U	1.3183E 01	1.4396E 01	1.5080E 01	1.5582E 01	1.5806E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01
	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01	1.5804E 01
T	3.8057E 02	3.8851E 02	3.9329E 02	3.9728E 02	3.9986E 02	4.0000E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02
	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9998E 02
RHO	6.4930E-01	6.3604E-01	6.2830E-01	6.2200E-01	6.1799E-01	6.1777E-01	6.1780E-01	6.1779E-01	6.1780E-01	6.1779E-01
	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1780E-01	6.1780E-01	6.1778E-01
H	3.8190E 05	3.8987E 05	3.9467E 05	3.9867E 05	4.0126E 05	4.0140E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05
	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0139E 05

DIST. ALONG X= 1.2000**NEAR WALL RE=1.3101E 03

Z	3.5501E-03	7.1089E-03	1.0980E-02	1.6594E-02	2.4024E-02	3.3427E-02	4.4973E-02	5.8850E-02	7.5264E-02	9.4390E-02
	1.1633E-01	1.4117E-01	1.6894E-01	1.9961E-01	2.3301E-01	2.6873E-01	3.0584E-01	3.4215E-01	3.7239E-01	5.0000E-01
U	1.1853E 01	1.3039E 01	1.3783E 01	1.4462E 01	1.5025E 01	1.5464E 01	1.5778E 01	1.5959E 01	1.5991E 01	1.5987E 01
	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01	1.5987E 01
T	3.7063E 02	3.7790E 02	3.8258E 02	3.8700E 02	3.9090E 02	3.9425E 02	3.9704E 02	3.9920E 02	4.0007E 02	3.9996E 02
	3.9998E 02	3.9997E 02	3.9998E 02	3.9998E 02	3.9998E 02	3.9997E 02	3.9997E 02	3.9997E 02	3.9997E 02	3.9999E 02
RHO	6.6670E-01	6.5388E-01	6.4588E-01	6.3849E-01	6.3212E-01	6.2676E-01	6.2235E-01	6.1899E-01	6.1764E-01	6.1781E-01
	6.1778E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1779E-01	6.1780E-01	6.1780E-01	6.1777E-01
H	3.7193E 05	3.7922E 05	3.8392E 05	3.8836E 05	3.9227E 05	3.9563E 05	3.9843E 05	4.0059E 05	4.0147E 05	4.0136E 05
	4.0138E 05	4.0137E 05	4.0138E 05	4.0138E 05	4.0138E 05	4.0137E 05	4.0137E 05	4.0137E 05	4.0137E 05	4.0139E 05

DIST. ALONG X= 2.2000**NEAR WALL RE=1.3194E 03

Z	3.6739E-03	7.3555E-03	1.1354E-02	1.7139E-02	2.4775E-02	3.4402E-02	4.6162E-02	6.0205E-02	7.6683E-02	9.5764E-02
	1.1762E-01	1.4236E-01	1.7003E-01	2.0058E-01	2.3385E-01	2.6943E-01	3.0639E-01	3.4253E-01	3.7261E-01	5.0000E-01
U	1.1331E 01	1.2481E 01	1.3214E 01	1.3900E 01	1.4495E 01	1.4996E 01	1.5407E 01	1.5727E 01	1.5955E 01	1.6085E 01
	1.6103E 01	1.6100E 01	1.6101E 01	1.6101E 01	1.6101E 01	1.6101E 01	1.6101E 01	1.6101E 01	1.6100E 01	1.6101E 01

```
T        3.6676E 02 3.7367E 02 3.7816E 02 3.8245E 02 3.8631E 02 3.8975E 02 3.9277E 02 3.9540E 02 3.9764E 02 3.9941E 02
         4.0005E 02 3.9996E 02 3.9997E 02 3.9997E 02 3.9997E 02 3.9997E 02 3.9996E 02 3.9995E 02 3.9996E 02 3.9999E 02

RHO      6.7373E-01 6.6126E-01 6.5342E-01 6.4609E-01 6.3962E-01 6.3399E-01 6.2911E-01 6.2492E-01 6.2140E-01 6.1866E-01
         6.1766E-01 6.1781E-01 6.1778E-01 6.1779E-01 6.1779E-01 6.1779E-01 6.1779E-01 6.1781E-01 6.1780E-01 6.1776E-01

H        3.6804E 05 3.7498E 05 3.7948E 05 3.8379E 05 3.8767E 05 3.9111E 05 3.9415E 05 3.9679E 05 3.9903E 05 4.0080E 05
         4.0145E 05 4.0136E 05 4.0137E 05 4.0137E 05 4.0137E 05 4.0137E 05 4.0136E 05 4.0135E 05 4.0136E 05 4.0139E 05

**DIST. ALONG X= 3.2000****NEAR WALL RE=1.3253E 03

Z        3.7517E-03 7.5114E-03 1.1591E-02 1.7489E-02 2.5263E-02 3.5049E-02 4.6982E-02 6.1193E-02 7.7813E-02 9.6977E-02
         1.1882E-01 1.4349E-01 1.7106E-01 2.0152E-01 2.3468E-01 2.7015E-01 3.0699E-01 3.4392E-01 3.7300E-01 5.0000E-01

U        1.1020E 01 1.2144E 01 1.2866E 01 1.3547E 01 1.4148E 01 1.4669E 01 1.5113E 01 1.5484E 01 1.5783E 01 1.6008E 01
         1.6153E 01 1.6206E 01 1.6201E 01 1.6202E 01 1.6202E 01 1.6202E 01 1.6202E 01 1.6201E 01 1.6202E 01 1.6202E 01

T        3.6431E 02 3.7099E 02 3.7533E 02 3.7951E 02 3.8329E 02 3.8670E 02 3.8975E 02 3.9249E 02 3.9493E 02 3.9706E 02
         3.9879E 02 4.0001E 02 3.9996E 02 3.9996E 02 3.9996E 02 3.9996E 02 3.9996E 02 3.9994E 02 3.9995E 02 3.9999E 02

RHO      6.7824E-01 6.6603E-01 6.5833E-01 6.5108E-01 6.4465E-01 6.3898E-01 6.3397E-01 6.2954E-01 6.2565E-01 6.2230E-01
         6.1960E-01 6.1771E-01 6.1779E-01 6.1778E-01 6.1779E-01 6.1779E-01 6.1780E-01 6.1782E-01 6.1780E-01 6.1775E-01

H        3.6559E 05 3.7229E 05 3.7664E 05 3.8084E 05 3.8463E 05 3.8805E 05 3.9112E 05 3.9387E 05 3.9632E 05 3.9845E 05
         4.0018E 05 4.0141E 05 4.0136E 05 4.0136E 05 4.0136E 05 4.0136E 05 4.0136E 05 4.0134E 05 4.0135E 05 4.0139E 05

**DIST. ALONG X= 4.2000****NEAR WALL RE=1.3297E 03

Z        3.8068E-03 7.6219E-03 1.1760E-02 1.7738E-02 2.5614E-02 3.5519E-02 4.7585E-02 6.1935E-02 7.8690E-02 9.7966E-02
         1.1988E-01 1.4453E-01 1.7203E-01 2.0239E-01 2.3546E-01 2.7082E-01 3.0756E-01 3.4347E-01 3.7337E-01 5.0000E-01

U        1.0805E 01 1.1910E 01 1.2622E 01 1.3298E 01 1.3899E 01 1.4427E 01 1.4886E 01 1.5281E 01 1.5615E 01 1.5889E 01
         1.6098E 01 1.6239E 01 1.6301E 01 1.6297E 01 1.6297E 01 1.6297E 01 1.6297E 01 1.6297E 01 1.6297E 01 1.6297E 01

T        3.6250E 02 3.6900E 02 3.7323E 02 3.7731E 02 3.8102E 02 3.8438E 02 3.8742E 02 3.9018E 02 3.9269E 02 3.9495E 02
         3.9696E 02 3.9863E 02 3.9993E 02 3.9996E 02 3.9995E 02 3.9995E 02 3.9995E 02 3.9993E 02 3.9995E 02 3.9998E 02

RHO      6.8161E-01 6.6962E-01 6.6203E-01 6.5487E-01 6.4849E-01 6.4282E-01 6.3778E-01 6.3327E-01 6.2922E-01 6.2561E-01
         6.2245E-01 6.1984E-01 6.1782E-01 6.1777E-01 6.1779E-01 6.1779E-01 6.1780E-01 6.1783E-01 6.1780E-01 6.1774E-01

H        3.6377E 05 3.7029E 05 3.7453E 05 3.7863E 05 3.8235E 05 3.8572E 05 3.8877E 05 3.9154E 05 3.9406E 05 3.9633E 05
         3.9834E 05 4.0003E 05 4.0133E 05 4.0136E 05 4.0135E 05 4.0135E 05 4.0135E 05 4.0133E 05 4.0135E 05 4.0138E 05

**DIST. ALONG X= 5.2000****NEAR WALL RE=1.3334E 03

Z        3.8491E-03 7.7072E-03 1.1890E-02 1.7930E-02 2.5884E-02 3.5883E-02 4.8054E-02 6.2518E-02 7.9389E-02 9.8774E-02
         1.2077E-01 1.4547E-01 1.7295E-01 2.0322E-01 2.3619E-01 2.7146E-01 3.0809E-01 3.4391E-01 3.7372E-01 5.0000E-01

U        1.0643E 01 1.1734E 01 1.2437E 01 1.3108E 01 1.3707E 01 1.4238E 01 1.4705E 01 1.5113E 01 1.5468E 01 1.5770E 01
         1.6018E 01 1.6209E 01 1.6337E 01 1.6392E 01 1.6388E 01 1.6389E 01 1.6389E 01 1.6388E 01 1.6388E 01 1.6388E 01

T        3.6105E 02 3.6740E 02 3.7154E 02 3.7553E 02 3.7918E 02 3.8249E 02 3.8550E 02 3.8826E 02 3.9079E 02 3.9311E 02
         3.9523E 02 3.9712E 02 3.9871E 02 3.9995E 02 3.9995E 02 3.9995E 02 3.9994E 02 3.9992E 02 3.9994E 02 3.9998E 02

RHO      6.8435E-01 6.7252E-01 6.6503E-01 6.5795E-01 6.5162E-01 6.4599E-01 6.4094E-01 6.3639E-01 6.3227E-01 6.2853E-01
         6.2516E-01 6.2218E-01 6.1970E-01 6.1779E-01 6.1778E-01 6.1779E-01 6.1780E-01 6.1783E-01 6.1780E-01 6.1773E-01

H        3.6231E 05 3.6868E 05 3.7284E 05 3.7685E 05 3.8051E 05 3.8383E 05 3.8685E 05 3.8961E 05 3.9215E 05 3.9449E 05
         3.9661E 05 3.9851E 05 4.0011E 05 4.0135E 05 4.0135E 05 4.0135E 05 4.0134E 05 4.0132E 05 4.0134E 05 4.0138E 05
```

DIST. ALONG X= 6.2000**NEAR WALL RE=1.3366E 03

Z	3.8830E-03	7.7748E-03	1.1993E-02	1.8083E-02	2.6099E-02	3.6173E-02	4.8431E-02	6.2989E-02	7.9958E-02	9.9440E-02
	1.2152E-01	1.4629E-01	1.7379E-01	2.0402E-01	2.3690E-01	2.7207E-01	3.0860E-01	3.4433E-01	3.7405E-01	5.0000E-01
U	1.0514E 01	1.1594E 01	1.2291E 01	1.2956E 01	1.3553E 01	1.4084E 01	1.4555E 01	1.4972E 01	1.5340E 01	1.5660E 01
	1.5933E 01	1.6158E 01	1.6329E 01	1.6441E 01	1.6480E 01	1.6476E 01	1.6476E 01	1.6476E 01	1.6476E 01	1.6476E 01
T	3.5982E 02	3.6605E 02	3.7011E 02	3.7403E 02	3.7762E 02	3.8088E 02	3.8386E 02	3.8660E 02	3.8914E 02	3.9148E 02
	3.9366E 02	3.9566E 02	3.9745E 02	3.9895E 02	4.0000E 02	3.9993E 02	3.9993E 02	3.9991E 02	3.9993E 02	3.9998E 02
RHO	6.8667E-01	6.7499E-01	6.6758E-01	6.6058E-01	6.5430E-01	6.4870E-01	6.4366E-01	6.3910E-01	6.3494E-01	6.3113E-01
	6.2765E-01	6.2448E-01	6.2165E-01	6.1932E-01	6.1770E-01	6.1780E-01	6.1780E-01	6.1784E-01	6.1780E-01	6.1772E-01
H	3.6108E 05	3.6733E 05	3.7140E 05	3.7534E 05	3.7894E 05	3.8222E 05	3.8521E 05	3.8796E 05	3.9050E 05	3.9285E 05
	3.9504E 05	3.9704E 05	3.9884E 05	4.0034E 05	4.0140E 05	4.0133E 05	4.0133E 05	4.0131E 05	4.0133E 05	4.0138E 05

DIST. ALONG X= 7.2000**NEAR WALL RE=1.3393E 03

Z	3.9104E-03	7.8297E-03	1.2077E-02	1.8207E-02	2.6275E-02	3.6410E-02	4.8739E-02	6.3377E-02	8.0430E-02	9.9997E-02
	1.2216E-01	1.4699E-01	1.7453E-01	2.0477E-01	2.3758E-01	2.7265E-01	3.0909E-01	3.4472E-01	3.7438E-01	5.0000E-01
U	1.0410E 01	1.1479E 01	1.2170E 01	1.2831E 01	1.3426E 01	1.3956E 01	1.4430E 01	1.4852E 01	1.5228E 01	1.5561E 01
	1.5852E 01	1.6099E 01	1.6301E 01	1.6451E 01	1.6544E 01	1.6564E 01	1.6561E 01	1.6561E 01	1.6561E 01	1.6561E 01
T	3.5876E 02	3.6487E 02	3.6887E 02	3.7273E 02	3.7626E 02	3.7948E 02	3.8242E 02	3.8514E 02	3.8767E 02	3.9003E 02
	3.9223E 02	3.9428E 02	3.9617E 02	3.9787E 02	3.9928E 02	4.0002E 02	3.9991E 02	3.9990E 02	3.9992E 02	3.9998E 02
RHO	6.8869E-01	6.7715E-01	6.6982E-01	6.6288E-01	6.5666E-01	6.5109E-01	6.4607E-01	6.4151E-01	6.3733E-01	6.3348E-01
	6.2992E-01	6.2665E-01	6.2366E-01	6.2099E-01	6.1880E-01	6.1765E-01	6.1782E-01	6.1784E-01	6.1780E-01	6.1771E-01
H	3.6001E 05	3.6615E 05	3.7016E 05	3.7403E 05	3.7757E 05	3.8080E 05	3.8376E 05	3.8649E 05	3.8903E 05	3.9139E 05
	3.9360E 05	3.9566E 05	3.9756E 05	3.9926E 05	4.0068E 05	4.0142E 05	4.0131E 05	4.0130E 05	4.0132E 05	4.0138E 05

DIST. ALONG X= 8.2000**NEAR WALL RE=1.3415E 03

Z	3.9331E-03	7.8755E-03	1.2147E-02	1.8312E-02	2.6423E-02	3.6610E-02	4.8999E-02	6.3704E-02	8.0830E-02	1.0047E-01
	1.2271E-01	1.4760E-01	1.7519E-01	2.0544E-01	2.3823E-01	2.7323E-01	3.0957E-01	3.4511E-01	3.7469E-01	5.0000E-01
U	1.0320E 01	1.1381E 01	1.2067E 01	1.2724E 01	1.3316E 01	1.3847E 01	1.4321E 01	1.4747E 01	1.5129E 01	1.5471E 01
	1.5775E 01	1.6039E 01	1.6263E 01	1.6442E 01	1.6570E 01	1.6640E 01	1.6645E 01	1.6643E 01	1.6643E 01	1.6643E 01
T	3.5781E 02	3.6383E 02	3.6776E 02	3.7156E 02	3.7504E 02	3.7822E 02	3.8113E 02	3.8383E 02	3.8634E 02	3.8870E 02
	3.9091E 02	3.9300E 02	3.9495E 02	3.9674E 02	3.9833E 02	3.9964E 02	3.9997E 02	3.9988E 02	3.9992E 02	3.9998E 02
RHO	6.9051E-01	6.7908E-01	6.7183E-01	6.6495E-01	6.5878E-01	6.5324E-01	6.4825E-01	6.4369E-01	6.3951E-01	6.3563E-01
	6.3203E-01	6.2868E-01	6.2557E-01	6.2275E-01	6.2027E-01	6.1823E-01	6.1772E-01	6.1786E-01	6.1780E-01	6.1770E-01
H	3.5906E 05	3.6510E 05	3.6905E 05	3.7286E 05	3.7635E 05	3.7954E 05	3.8247E 05	3.8517E 05	3.8769E 05	3.9006E 05
	3.9228E 05	3.9437E 05	3.9633E 05	3.9813E 05	3.9972E 05	4.0104E 05	4.0137E 05	4.0128E 05	4.0132E 05	4.0138E 05

DIST. ALONG X= 9.2000**NEAR WALL RE=1.3438E 03

Z	3.9528E-03	7.9147E-03	1.2207E-02	1.8401E-02	2.6548E-02	3.6780E-02	4.9221E-02	6.3994E-02	8.1172E-02	1.0088E-01
	1.2318E-01	1.4813E-01	1.7577E-01	2.0605E-01	2.3884E-01	2.7378E-01	3.1003E-01	3.4549E-01	3.7499E-01	5.0000E-01
U	1.0244E 01	1.1298E 01	1.1979E 01	1.2633E 01	1.3222E 01	1.3751E 01	1.4227E 01	1.4655E 01	1.5042E 01	1.5390E 01
	1.5703E 01	1.5981E 01	1.6221E 01	1.6422E 01	1.6578E 01	1.6683E 01	1.6727E 01	1.6723E 01	1.6723E 01	1.6724E 01
T	3.5695E 02	3.6288E 02	3.6675E 02	3.7050E 02	3.7394E 02	3.7707E 02	3.7996E 02	3.8263E 02	3.8513E 02	3.8747E 02
	3.8969E 02	3.9180E 02	3.9379E 02	3.9565E 02	3.9734E 02	3.9879E 02	3.9989E 02	3.9988E 02	3.9991E 02	3.9998E 02

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RHO     6.9215E-01 6.8084E-01 6.7365E-01 6.6684E-01 6.6071E-01 6.5521E-01 6.5024E-01 6.4570E-01 6.4152E-01 6.3763E-01
        6.3400E-01 6.3059E-01 6.2741E-01 6.2446E-01 6.2179E-01 6.1954E-01 6.1782E-01 6.1784E-01 6.1781E-01 6.1769E-01

H       3.5820E 05 3.6415E 05 3.6804E 05 3.7180E 05 3.7525E 05 3.7839E 05 3.8129E 05 3.8397E 05 3.8647E 05 3.8893E 05
        3.9106E 05 3.9317E 05 3.9516E 05 3.9703E 05 3.9873E 05 4.0019E 05 4.0129E 05 4.0129E 05 4.0131E 05 4.0138E 05

**DIST. ALONG X=10.1999****NEAR WALL RE=1.3458E 03

Z       3.9696E-03 7.9487E-03 1.2259E-02 1.8478E-02 2.6657E-02 3.6926E-02 4.9411E-02 6.4224E-02 8.1467E-02 1.0123E-01
        1.2359E-01 1.4859E-01 1.7628E-01 2.0659E-01 2.3939E-01 2.7431E-01 3.1049E-01 3.4585E-01 3.7529E-01 5.0000E-01

U       1.0178E 01 1.1225E 01 1.1903E 01 1.2553E 01 1.3140E 01 1.3668E 01 1.4143E 01 1.4573E 01 1.4963E 01 1.5317E 01
        1.5637E 01 1.5925E 01 1.6178E 01 1.6396E 01 1.6575E 01 1.6707E 01 1.6786E 01 1.6806E 01 1.6802E 01 1.6803E 01

T       3.5616E 02 3.6201E 02 3.6583E 02 3.6953E 02 3.7292E 02 3.7602E 02 3.7887E 02 3.8152E 02 3.8400E 02 3.8634E 02
        3.8855E 02 3.9067E 02 3.9268E 02 3.9458E 02 3.9635E 02 3.9795E 02 3.9925E 02 3.9995E 02 3.9989E 02 3.9999E 02

RHO     6.9367E-01 6.8246E-01 6.7534E-01 6.6858E-01 6.6250E-01 6.5704E-01 6.5209E-01 6.4757E-01 6.4339E-01 6.3949E-01
        6.3584E-01 6.3241E-01 6.2917E-01 6.2614E-01 6.2334E-01 6.2084E-01 6.1882E-01 6.1773E-01 6.1782E-01 6.1768E-01

H       3.5741E 05 3.6328E 05 3.6711E 05 3.7082E 05 3.7423E 05 3.7734E 05 3.8020E 05 3.8286E 05 3.8534E 05 3.8769E 05
        3.8991E 05 3.9203E 05 3.9405E 05 3.9596E 05 3.9773E 05 3.9934E 05 4.0064E 05 4.0135E 05 4.0129E 05 4.0138E 05
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EX	PRESS	P-RECV	WALL SHEAR (Pa)	HEAT FLX (W/m²)	B.L. THK	DISP THK	MOM THK	SHAPE FAC
0.0	7.093E 04	0.0	4.072E-01	3.892E 03	1.018E-02	1.448E-03	1.141E-03	1.269E 00
3.906E-04	7.093E 04	-2.432E-02	4.255E-01	3.978E 03	1.002E-02	1.340E-03	1.100E-03	1.254E 00
7.812E-04	7.092E 04	-4.505E-02	4.407E-01	4.048E 03	9.893E-03	1.331E-03	1.072E-03	1.242E 00
1.562E-03	7.092E 04	-6.126E-02	4.526E-01	4.100E 03	9.819E-03	1.295E-03	1.051E-03	1.232E 00
3.125E-03	7.092E 04	-7.387E-02	4.612E-01	4.136E 03	9.809E-03	1.270E-03	1.039E-03	1.223E 00
6.250E-03	7.092E 04	-8.378E-02	4.668E-01	4.156E 03	9.888E-03	1.256E-03	1.032E-03	1.216E 00
1.250E-02	7.092E 04	-9.189E-02	4.692E-01	4.157E 03	1.132E-02	1.255E-03	1.040E-03	1.208E 00
2.500E-02	7.092E 04	-9.820E-02	4.682E-01	4.138E 03	1.266E-02	1.276E-03	1.069E-03	1.194E 00
5.000E-02	7.092E 04	-1.036E-01	4.628E-01	4.087E 03	1.360E-02	1.333E-03	1.138E-03	1.172E 00
1.000E-01	7.092E 04	-1.081E-01	4.515E-01	3.978E 03	1.443E-02	1.454E-03	1.280E-03	1.135E 00
2.000E-01	7.092E 04	-1.135E-01	4.324E-01	3.788E 03	2.005E-02	1.693E-03	1.563E-03	1.083E 00
3.000E-01	7.092E 04	-1.180E-01	4.183E-01	3.647E 03	2.250E-02	1.919E-03	1.834E-03	1.046E 00
4.000E-01	7.092E 04	-1.216E-01	4.069E-01	3.534E 03	2.789E-02	2.139E-03	2.099E-03	1.019E 00
5.000E-01	7.092E 04	-1.252E-01	3.974E-01	3.441E 03	3.064E-02	2.352E-03	2.355E-03	9.985E-01
6.000E-01	7.092E 04	-1.279E-01	3.894E-01	3.362E 03	3.407E-02	2.560E-03	2.607E-03	9.819E-01
7.000E-01	7.092E 04	-1.306E-01	3.824E-01	3.294E 03	3.866E-02	2.763E-03	2.854E-03	9.681E-01
8.000E-01	7.092E 04	-1.333E-01	3.762E-01	3.234E 03	4.145E-02	2.963E-03	3.098E-03	9.564E-01
9.000E-01	7.092E 04	-1.360E-01	3.707E-01	3.181E 03	4.387E-02	3.159E-03	3.337E-03	9.465E-01
1.000E 00	7.092E 04	-1.387E-01	3.658E-01	3.134E 03	4.854E-02	3.352E-03	3.574E-03	9.379E-01
1.100E 00	7.092E 04	-1.414E-01	3.613E-01	3.090E 03	5.213E-02	3.542E-03	3.808E-03	9.303E-01
1.200E 00	7.092E 04	-1.432E-01	3.572E-01	3.051E 03	5.489E-02	3.730E-03	4.039E-03	9.237E-01
1.300E 00	7.092E 04	-1.450E-01	3.534E-01	3.015E 03	5.735E-02	3.916E-03	4.268E-03	9.177E-01
1.400E 00	7.092E 04	-1.468E-01	3.499E-01	2.981E 03	6.045E-02	4.101E-03	4.494E-03	9.124E-01
1.500E 00	7.092E 04	-1.486E-01	3.467E-01	2.950E 03	6.488E-02	4.283E-03	4.720E-03	9.075E-01
1.600E 00	7.092E 04	-1.505E-01	3.437E-01	2.922E 03	6.808E-02	4.463E-03	4.943E-03	9.030E-01
1.700E 00	7.092E 04	-1.523E-01	3.409E-01	2.894E 03	7.079E-02	4.642E-03	5.164E-03	8.990E-01
1.800E 00	7.092E 04	-1.541E-01	3.383E-01	2.869E 03	7.328E-02	4.820E-03	5.384E-03	8.952E-01
1.900E 00	7.092E 04	-1.559E-01	3.358E-01	2.845E 03	7.557E-02	4.995E-03	5.602E-03	8.917E-01
2.000E 00	7.092E 04	-1.577E-01	3.335E-01	2.822E 03	7.945E-02	5.169E-03	5.818E-03	8.884E-01
2.100E 00	7.092E 04	-1.595E-01	3.313E-01	2.801E 03	8.318E-02	5.342E-03	6.034E-03	8.854E-01
2.200E 00	7.092E 04	-1.613E-01	3.292E-01	2.780E 03	8.623E-02	5.514E-03	6.248E-03	8.825E-01
2.300E 00	7.092E 04	-1.631E-01	3.272E-01	2.761E 03	8.894E-02	5.684E-03	6.461E-03	8.799E-01
2.400E 00	7.092E 04	-1.649E-01	3.253E-01	2.742E 03	9.143E-02	5.854E-03	6.672E-03	8.774E-01
2.500E 00	7.092E 04	-1.667E-01	3.235E-01	2.725E 03	9.376E-02	6.023E-03	6.883E-03	8.750E-01
2.600E 00	7.092E 04	-1.685E-01	3.218E-01	2.708E 03	9.592E-02	6.189E-03	7.092E-03	8.727E-01
2.700E 00	7.092E 04	-1.703E-01	3.202E-01	2.692E 03	9.982E-02	6.356E-03	7.301E-03	8.706E-01
2.800E 00	7.092E 04	-1.721E-01	3.187E-01	2.676E 03	1.033E-01	6.521E-03	7.508E-03	8.685E-01
2.900E 00	7.092E 04	-1.739E-01	3.172E-01	2.661E 03	1.063E-01	6.684E-03	7.714E-03	8.665E-01
3.000E 00	7.092E 04	-1.757E-01	3.157E-01	2.646E 03	1.090E-01	6.848E-03	7.920E-03	8.646E-01
3.100E 00	7.092E 04	-1.775E-01	3.143E-01	2.632E 03	1.116E-01	7.011E-03	8.125E-03	8.629E-01
3.200E 00	7.092E 04	-1.793E-01	3.130E-01	2.619E 03	1.140E-01	7.172E-03	8.329E-03	8.611E-01
3.300E 00	7.091E 04	-1.811E-01	3.118E-01	2.606E 03	1.162E-01	7.334E-03	8.533E-03	8.595E-01
3.400E 00	7.091E 04	-1.829E-01	3.105E-01	2.593E 03	1.183E-01	7.494E-03	8.735E-03	8.579E-01
3.500E 00	7.091E 04	-1.847E-01	3.094E-01	2.581E 03	1.215E-01	7.654E-03	8.937E-03	8.565E-01
3.600E 00	7.091E 04	-1.865E-01	3.082E-01	2.570E 03	1.250E-01	7.813E-03	9.139E-03	8.550E-01
3.700E 00	7.091E 04	-1.883E-01	3.071E-01	2.558E 03	1.281E-01	7.971E-03	9.339E-03	8.535E-01
3.800E 00	7.091E 04	-1.901E-01	3.061E-01	2.547E 03	1.309E-01	8.129E-03	9.539E-03	8.522E-01
3.900E 00	7.091E 04	-1.919E-01	3.050E-01	2.536E 03	1.335E-01	8.287E-03	9.739E-03	8.509E-01
4.000E 00	7.091E 04	-1.937E-01	3.040E-01	2.526E 03	1.360E-01	8.443E-03	9.938E-03	8.496E-01
4.100E 00	7.091E 04	-1.955E-01	3.031E-01	2.516E 03	1.383E-01	8.599E-03	1.014E-02	8.484E-01
4.200E 00	7.091E 04	-1.973E-01	3.021E-01	2.506E 03	1.405E-01	8.755E-03	1.033E-02	8.472E-01
4.300E 00	7.091E 04	-1.991E-01	3.012E-01	2.496E 03	1.425E-01	8.910E-03	1.053E-02	8.460E-01
4.400E 00	7.091E 04	-2.009E-01	3.003E-01	2.486E 03	1.445E-01	9.064E-03	1.073E-02	8.449E-01
4.500E 00	7.091E 04	-2.027E-01	2.995E-01	2.477E 03	1.479E-01	9.218E-03	1.092E-02	8.438E-01
4.600E 00	7.091E 04	-2.045E-01	2.987E-01	2.468E 03	1.512E-01	9.372E-03	1.112E-02	8.428E-01
4.700E 00	7.091E 04	-2.063E-01	2.978E-01	2.459E 03	1.541E-01	9.525E-03	1.132E-02	8.417E-01
4.800E 00	7.091E 04	-2.081E-01	2.971E-01	2.451E 03	1.568E-01	9.677E-03	1.151E-02	8.407E-01
4.900E 00	7.091E 04	-2.099E-01	2.963E-01	2.442E 03	1.594E-01	9.829E-03	1.170E-02	8.397E-01
5.000E 00	7.091E 04	-2.117E-01	2.955E-01	2.434E 03	1.618E-01	9.981E-03	1.190E-02	8.388E-01
5.100E 00	7.091E 04	-2.135E-01	2.948E-01	2.426E 03	1.641E-01	1.013E-02	1.209E-02	8.379E-01

5.200E 00	7.091E 04	-2.153E-01	2.941E-01	2.418E 03	1.663E-01	1.028E-02	1.229E-02	8.370E-01
5.300E 00	7.091E 04	-2.171E-01	2.934E-01	2.411E 03	1.694E-01	1.044E-02	1.248E-02	8.361E-01
5.400E 00	7.091E 04	-2.189E-01	2.927E-01	2.403E 03	1.704E-01	1.059E-02	1.267E-02	8.353E-01
5.500E 00	7.091E 04	-2.207E-01	2.921E-01	2.396E 03	1.723E-01	1.074E-02	1.287E-02	8.344E-01
5.600E 00	7.091E 04	-2.225E-01	2.914E-01	2.388E 03	1.750E-01	1.089E-02	1.306E-02	8.336E-01
5.700E 00	7.091E 04	-2.243E-01	2.908E-01	2.381E 03	1.782E-01	1.103E-02	1.325E-02	8.328E-01
5.800E 00	7.091E 04	-2.261E-01	2.902E-01	2.374E 03	1.812E-01	1.118E-02	1.344E-02	8.320E-01
5.900E 00	7.091E 04	-2.279E-01	2.896E-01	2.367E 03	1.839E-01	1.133E-02	1.363E-02	8.312E-01
6.000E 00	7.091E 04	-2.297E-01	2.890E-01	2.361E 03	1.865E-01	1.148E-02	1.382E-02	8.305E-01
6.100E 00	7.091E 04	-2.315E-01	2.884E-01	2.354E 03	1.889E-01	1.163E-02	1.402E-02	8.297E-01
6.200E 00	7.091E 04	-2.333E-01	2.878E-01	2.348E 03	1.913E-01	1.178E-02	1.421E-02	8.290E-01
6.300E 00	7.091E 04	-2.351E-01	2.873E-01	2.341E 03	1.935E-01	1.192E-02	1.440E-02	8.283E-01
6.400E 00	7.091E 04	-2.369E-01	2.867E-01	2.335E 03	1.957E-01	1.207E-02	1.459E-02	8.275E-01
6.500E 00	7.091E 04	-2.387E-01	2.862E-01	2.329E 03	1.977E-01	1.222E-02	1.478E-02	8.269E-01
6.600E 00	7.091E 04	-2.405E-01	2.857E-01	2.323E 03	1.997E-01	1.236E-02	1.497E-02	8.262E-01
6.700E 00	7.091E 04	-2.423E-01	2.852E-01	2.317E 03	2.016E-01	1.251E-02	1.515E-02	8.255E-01
6.800E 00	7.091E 04	-2.441E-01	2.847E-01	2.311E 03	2.034E-01	1.266E-02	1.534E-02	8.248E-01
6.900E 00	7.091E 04	-2.459E-01	2.842E-01	2.305E 03	2.058E-01	1.280E-02	1.553E-02	8.242E-01
7.000E 00	7.091E 04	-2.477E-01	2.837E-01	2.299E 03	2.089E-01	1.295E-02	1.572E-02	8.236E-01
7.100E 00	7.091E 04	-2.495E-01	2.833E-01	2.294E 03	2.117E-01	1.309E-02	1.591E-02	8.229E-01
7.200E 00	7.091E 04	-2.514E-01	2.828E-01	2.288E 03	2.144E-01	1.324E-02	1.610E-02	8.223E-01
7.300E 00	7.091E 04	-2.532E-01	2.823E-01	2.283E 03	2.170E-01	1.338E-02	1.629E-02	8.217E-01
7.400E 00	7.091E 04	-2.550E-01	2.819E-01	2.277E 03	2.194E-01	1.353E-02	1.648E-02	8.211E-01
7.500E 00	7.091E 04	-2.568E-01	2.814E-01	2.272E 03	2.217E-01	1.367E-02	1.666E-02	8.205E-01
7.600E 00	7.091E 04	-2.586E-01	2.810E-01	2.267E 03	2.240E-01	1.382E-02	1.685E-02	8.199E-01
7.700E 00	7.091E 04	-2.604E-01	2.806E-01	2.262E 03	2.261E-01	1.396E-02	1.704E-02	8.194E-01
7.800E 00	7.091E 04	-2.622E-01	2.802E-01	2.257E 03	2.282E-01	1.411E-02	1.723E-02	8.188E-01
7.900E 00	7.091E 04	-2.640E-01	2.798E-01	2.252E 03	2.302E-01	1.425E-02	1.742E-02	8.183E-01
8.000E 00	7.091E 04	-2.658E-01	2.793E-01	2.247E 03	2.321E-01	1.440E-02	1.760E-02	8.178E-01
8.100E 00	7.091E 04	-2.676E-01	2.789E-01	2.242E 03	2.339E-01	1.454E-02	1.779E-02	8.172E-01
8.200E 00	7.091E 04	-2.694E-01	2.786E-01	2.237E 03	2.357E-01	1.468E-02	1.798E-02	8.167E-01
8.300E 00	7.091E 04	-2.712E-01	2.782E-01	2.232E 03	2.375E-01	1.483E-02	1.817E-02	8.162E-01
8.400E 00	7.091E 04	-2.730E-01	2.778E-01	2.227E 03	2.399E-01	1.497E-02	1.835E-02	8.157E-01
8.500E 00	7.091E 04	-2.748E-01	2.774E-01	2.223E 03	2.428E-01	1.511E-02	1.854E-02	8.152E-01
8.600E 00	7.091E 04	-2.766E-01	2.771E-01	2.218E 03	2.455E-01	1.526E-02	1.873E-02	8.147E-01
8.700E 00	7.091E 04	-2.784E-01	2.767E-01	2.214E 03	2.481E-01	1.540E-02	1.891E-02	8.142E-01
8.800E 00	7.091E 04	-2.802E-01	2.764E-01	2.209E 03	2.506E-01	1.554E-02	1.910E-02	8.137E-01
8.900E 00	7.091E 04	-2.820E-01	2.760E-01	2.205E 03	2.530E-01	1.568E-02	1.929E-02	8.132E-01
9.000E 00	7.091E 04	-2.838E-01	2.757E-01	2.200E 03	2.553E-01	1.583E-02	1.947E-02	8.127E-01
9.100E 00	7.091E 04	-2.856E-01	2.753E-01	2.196E 03	2.574E-01	1.597E-02	1.966E-02	8.122E-01
9.200E 00	7.091E 04	-2.874E-01	2.750E-01	2.192E 03	2.596E-01	1.611E-02	1.985E-02	8.117E-01
9.300E 00	7.091E 04	-2.892E-01	2.747E-01	2.187E 03	2.616E-01	1.625E-02	2.003E-02	8.112E-01
9.400E 00	7.091E 04	-2.910E-01	2.743E-01	2.183E 03	2.636E-01	1.639E-02	2.022E-02	8.107E-01
9.500E 00	7.091E 04	-2.928E-01	2.740E-01	2.179E 03	2.655E-01	1.653E-02	2.041E-02	8.103E-01
9.600E 00	7.091E 04	-2.946E-01	2.737E-01	2.175E 03	2.673E-01	1.668E-02	2.059E-02	8.098E-01
9.700E 00	7.091E 04	-2.964E-01	2.734E-01	2.171E 03	2.691E-01	1.682E-02	2.078E-02	8.093E-01
9.800E 00	7.091E 04	-2.982E-01	2.731E-01	2.167E 03	2.709E-01	1.696E-02	2.097E-02	8.089E-01
9.900E 00	7.091E 04	-3.000E-01	2.728E-01	2.163E 03	2.726E-01	1.710E-02	2.115E-02	8.084E-01
1.000E 01	7.091E 04	-3.018E-01	2.725E-01	2.159E 03	2.743E-01	1.724E-02	2.134E-02	8.080E-01
1.010E 01	7.091E 04	-3.036E-01	2.722E-01	2.155E 03	2.771E-01	1.738E-02	2.152E-02	8.075E-01
1.020E 01	7.091E 04	-3.054E-01	2.719E-01	2.151E 03	2.798E-01	1.752E-02	2.171E-02	8.071E-01

3.5 Listing of RECTANGULAR Geometry Subroutines.

For rectangular ducts the computations are carried out along one pair of opposing walls. Half the distance between these two walls is called HEIGHT. Half the distance between the other pair of opposing walls is called WIDTH. The effect of the WIDTH walls is taken into account, as in Refs. 5 and 9, by using effective width, EFWIDH, in calculations involving mass flow rates. Effective width is calculated as,

$$\text{EFWIDH} = \text{WIDTH} - \delta_1 \qquad \text{for } \delta_0 \leq \text{WIDTH}$$

$$\text{EFWIDH} = \text{WIDTH}\left(1 - \frac{\delta_1}{\delta_0}\right) \qquad \text{for } \delta_0 > \text{WIDTH}$$

where δ_0 and δ_1 are the boundary layer thickness and the displacement thickness computed along HEIGHT walls.

It is assumed that the flow is symmetric about the midplanes. The computations are carried out from wall to half the height.

INPUT1 Subroutine:

This subroutine provides data for the duct shape.

WIDTH = Half the duct width,
HEIGHT = Half the duct height,
WSLOPE,HSLOPE = Slope of the duct walls defined as:
WIDTH at $x + \Delta x$ = WIDTH at $x +$ WSLOPE$\cdot\Delta x$
HEIGHT at $x + \Delta x$ = HEIGHT at $x +$ HSLOPE$\cdot\Delta x$
ARFAC = 1 for rectangular ducts

SIPSIM and ZDZ Subroutines:

Given U(J), RHO(J), and Z(J) subroutine SIPSIM calculates SIP(J) and SIM(J). It is called at the beginning of each integration step.

Subroutine ZDZ, called at the end of the integration step, calculates Z(J) given U(J), RHO(J), SIP(J), and SIM(J). Width used in these calculations is the effective width, EFWIDH. Fortran statements in these routiens are almost direct translations of the formulae given in Appendices A and B of Ref. 7.

XSAREA Subroutine:

This subroutine calculates cross-sectional and surface areas of the streams. It also calculates the cross-sectional area of the duct called OLDA. This area is used in the pressure iteration formulae.

```
      SUBROUTINE INPUTI
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,CLDA,ARFAC
      WIDTH=[redacted]
      HEIGHT=[redacted]
      WSLOPE=[redacted]
      HSLOPE=[redacted]
      ARFAC=1.
      RETURN
      END
```

```
      SUBROUTINE SIPSIM(NP)
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM3/SIP(50),SIM(50),FLORAT
      COMMON/COM5/AX(50),AS(50)
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM7/DEL1,DEL2,H12,DELO
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,CLDA,ARFAC
      DIMENSION RU(50)
      IF(CLDA.LE.0.0)EFWIDH=WIDTH-DEL1
      WIDTT=EFWIDH*0.03125
      FLORAT=0.0
C**SI'S,EXCEPT SIM(2), REDUCED BY 4 TO ABSORV 1/4 APPEARING IN SI(J)/4**
      DO 10 J=2,NJ
   10 RU(J)=RHO(J)*U(J)
      DO 30 J=2,NJ1
      SIP(J)=(3.*RU(J)+RU(J+1))*DZ(J+1)*WIDTT
   30 FLORAT=FLORAT+SIP(J)
      SIM(2)=RU(2)*GAMMA*DZ(2)*WIDTT*32.
      FLORAT=FLORAT+SIM(2)*.25
      DO 40 J=3,NJ
      SIM(J)=(3.*RU(J)+RU(J-1))*DZ(J)*WIDTT
   40 FLORAT=FLORAT+SIM(J)
      FLORAT=FLORAT*4.0
      IF (NP.NE.1) GO TO 100
      WRITE(6,22) ( SIM(J),J=2,NJ)
      WRITE(6,24) ( SIP(J),J=2,NJ1)
      WRITE(6,53) FLORAT
   22 FORMAT(/,' SIM',6X  ,10E11.3/(10X,10E11.3))
   24 FORMAT(/,' SIP',6X  ,10E11.3/(10X,10E11.3))
   53 FORMAT (1X,9X,'FLOW RATE= ' , E11.4)
  100 RETURN
      END
```

S FORTRAN IV 360N-FU-479 3-8 ZDZ DATE 08/20/79 TIME 14

```
      SUBROUTINE ZDZ
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM3/SIP(50),SIM(50),FLORAT
      COMMON/COM6/ALPHA,BETA,DELTA,GAMMA,RE,VISC(50)
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,AKFAC
      DIMENSION RU(50)
      WINV= 8.0/EFWIDH
      DO 10 J=2,NJ
   10 RU(J)=RHO(J)*U(J)
      Z(2)=.125*WINV*SIM(2)/RU(2)/GAMMA
      DZ(2)=Z(2)
      DO 20 J=3,NJ
      DZ(J)=(SIP(J-1)+SIM(J))/(RU(J)+RU(J-1))*WINV
   20 Z(J)=Z(J-1)+DZ(J)
      RETURN
      END
```

```
      SUBROUTINE XSAREA(NP)
      COMMON/COM1/U(50),T(50),H(50),RHO(50),USH(50),Z(50),DZ(50)
      COMMON/COM2/NJ,NJ1,NJ2,NJP,NBL,DX
      COMMON/COM5/AX(50),AS(50)
      COMMON/COM7/DEL1,DEL2,H12,DELO
      COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,AVWIDH,EFWIDH,OLDA,ARFAC
C**CALCULATION OF NEW DUCT CROSS-SECTION****RRRRRRRRRRRRRRRRRRRRRRRRR
      HEIGHT=HEIGHT+HSLOPE*DX
      WIDTH=WIDTH+WSLOPE*DX
      AVWIDH=WIDTH+WSLOPE*DX*0.5
      WIDCOR=DEL1
      IF(DELO.GT.WIDTH) WIDCOR =WIDTH*DEL1/DELO
      EFWIDH=WIDTH-WIDCOR
      OLDA =HEIGHT*EFWIDH
      WHF=0.5*AVWIDH
      AX(2)=WHF*(Z(3)+Z(2))
      DO 10 J=3,NJ1
   10 AX(J)=WHF*(Z(J+1)-Z(J-1))
      AX(NJ)=WHF*(Z(NJ)-Z(NJ1))
      DO 20 J=1,NJ
   20 AS(J)=DX*AVWIDH
      IF(NP.NE.1)GO TO 35
      WRITE(6,30) (AS(J),J=2,NJ)
   30 FORMAT(/,' AS',7X,10E11.3/(10X,10E11.3))
      WRITE (6,32) (AX(J),J=2,NJ)
   32 FORMAT (/,' AX',7X,10E11.3/(10X,10E11.3))
   35 RETURN
      END
```

3.6 Sample Computations--Rectangular Diffuser Flow

DATA: (SI Units)

MAIN Program:

```
C**DYNAMIC AND THERMODYNAMIC DATA*****************************
      UCNTR=200.0
      ZBL=0.015
      TCNTR=300.0
      TWALL=300.0
      P=71000.0
      CP=1003.5
      GASK=1.4
      GASR=287.0
      PRNDL=0.7
      DPDX=210000.0
C**GEOMETRIC DUCT DATA****************************************
      CALL INPUT1
      XEND=0.5
C**CONVERGENCE CRITERIA,GRID SIZE,AND OTHER DATA**************
      DPDTL=0.01
      DXFAC=5.0
      MAXIT=50
      NJ=21
      NPRNT=5
C**DATA FOR VSCSTY AND START SUBROUTINES**********************
      VZERO=0.0000386
      TZERO=860.0
      EXPV=.7
      EXPU=7.
      EXPH=7.
C
```

INPUT1 Subroutine:

```
WIDTH=0.025
HEIGHT=0.05
WSLOPE=0.03
HSLOPE=0.03
```

JOB STATISTICS:

Machine: IBM 360/44 with DOS operating system.
Cards Read: 491
Core Required: 42K
Compilation Time: 3.05 mns
CPU Time: 1.97 mns

VELOCITY U,TEMPERATURE T,DENSITY RHO, AND ENTHAPY H DISTRIBUTIONS AS FUNCTION OF Z.

**DIST. ALONG X= 0.0 ****NEAR WALL RE=1.2000E 03

Z	2.4227E-04	4.8453E-04	7.7459E-04	1.2226E-03	1.8568E-03	2.7040E-03	3.7899E-03	5.1390E-03	6.7754E-03	8.7222E-03
	1.1002E-02	1.3637E-02	1.6648E-02	2.0057E-02	2.3884E-02	2.8151E-02	3.2876E-02	3.8079E-02	4.3781E-02	5.0000E-02
U	1.1093E 02	1.2248E 02	1.3097E 02	1.3979E 02	1.4839E 02	1.5658E 02	1.6432E 02	1.7162E 02	1.7853E 02	1.8509E 02
	1.9134E 02	1.9730E 02	2.0000E 02	2.0000E 02	2.0000E 02	2.0000E 02	2.0000E 02	2.0000E 02	2.0000E 02	2.0000E 02
T	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02
	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02	3.0000E 02
RHO	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01
	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01	8.2462E-01
H	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05
	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05	3.0105E 05

DIST. ALONG X= 0.0006**NEAR WALL RE=1.1988E 03

Z	2.4346E-04	4.8646E-04	7.7689E-04	1.2254E-03	1.8603E-03	2.7083E-03	3.7951E-03	5.1452E-03	6.7825E-03	8.7303E-03
	1.1011E-02	1.3647E-02	1.6660E-02	2.0070E-02	2.3898E-02	2.8165E-02	3.2891E-02	3.8096E-02	4.3799E-02	5.0019E-02
U	1.1034E 02	1.2219E 02	1.3068E 02	1.3949E 02	1.4809E 02	1.5628E 02	1.6402E 02	1.7134E 02	1.7826E 02	1.8482E 02
	1.9109E 02	1.9696E 02	1.9974E 02	1.9975E 02	1.9975E 02	1.9975E 02	1.9975E 02	1.9975E 02	1.9975E 02	1.9975E 02
T	3.0020E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02	3.0005E 02
	3.0005E 02	3.0005E 02	3.0005E 02	3.0004E 02	3.0005E 02	3.0004E 02	3.0004E 02	3.0004E 02	3.0004E 02	3.0005E 02
RHO	8.2450E-01	8.2491E-01	8.2492E-01	8.2492E-01	8.2492E-01	8.2492E-01	8.2492E-01	8.2492E-01	8.2492E-01	8.2492E-01
	8.2492E-01	8.2492E-01	8.2492E-01	8.2493E-01	8.2493E-01	8.2493E-01	8.2493E-01	8.2493E-01	8.2493E-01	8.2492E-01
H	3.0125E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05	3.0110E 05
	3.0110E 05	3.0110E 05	3.0110E 05	3.0109E 05	3.0110E 05	3.0109E 05	3.0109E 05	3.0109E 05	3.0109E 05	3.0110E 05

DIST. ALONG X= 0.0200**NEAR WALL RE=1.1707E 03

Z	2.6072E-04	5.1872E-04	8.2382E-04	1.2912E-03	1.9485E-03	2.8220E-03	3.9367E-03	5.3165E-03	6.9844E-03	8.9632E-03
	1.1277E-02	1.3953E-02	1.7008E-02	2.0448E-02	2.4301E-02	2.8598E-02	3.3356E-02	3.8597E-02	4.4339E-02	5.0601E-02
U	1.0061E 02	1.1271E 02	1.2151E 02	1.3054E 02	1.3931E 02	1.4765E 02	1.5555E 02	1.6303E 02	1.7013E 02	1.7680E 02
	1.8289E 02	1.8808E 02	1.9174E 02	1.9268E 02	1.9255E 02	1.9257E 02	1.9257E 02	1.9257E 02	1.9257E 02	1.9257E 02
T	3.0282E 02	3.0235E 02	3.0204E 02	3.0179E 02	3.0163E 02	3.0155E 02	3.0152E 02	3.0150E 02	3.0149E 02	3.0148E 02
	3.0147E 02	3.0146E 02	3.0145E 02	3.0144E 02	3.0144E 02	3.0144E 02	3.0144E 02	3.0144E 02	3.0144E 02	3.0146E 02
RHO	8.2966E-01	8.3097E-01	8.3182E-01	8.3251E-01	8.3294E-01	8.3315E-01	8.3325E-01	8.3330E-01	8.3333E-01	8.3336E-01
	8.3338E-01	8.3341E-01	8.3343E-01	8.3346E-01	8.3345E-01	8.3346E-01	8.3345E-01	8.3345E-01	8.3345E-01	8.3342E-01
H	3.0388E 05	3.0341E 05	3.0309E 05	3.0284E 05	3.0269E 05	3.0261E 05	3.0257E 05	3.0256E 05	3.0254E 05	3.0253E 05
	3.0253E 05	3.0252E 05	3.0251E 05	3.0250E 05	3.0250E 05	3.0250E 05	3.0250E 05	3.0250E 05	3.0250E 05	3.0251E 05

DIST. ALONG X= 0.0700**NEAR WALL RE=1.1247E 03

Z	2.8223E-04	5.6052E-04	8.8928E-04	1.3916E-03	2.0953E-03	3.0258E-03	4.2071E-03	5.6617E-03	7.4121E-03	9.4810E-03
	1.1893E-02	1.4676E-02	1.7839E-02	2.1375E-02	2.5307E-02	2.9683E-02	3.4532E-02	3.9871E-02	4.5721E-02	5.2102E-02
U	8.8177E 01	9.8914E 01	1.0689E 02	1.1524E 02	1.2352E 02	1.3156E 02	1.3931E 02	1.4670E 02	1.5369E 02	1.6022E 02
	1.6616E 02	1.7134E 02	1.7548E 02	1.7819E 02	1.7882E 02	1.7873E 02	1.7874E 02	1.7874E 02	1.7874E 02	1.7874E 02

```
T        3.0534E 02 3.0519E 02 3.0503E 02 3.0485E 02 3.0467E 02 3.0450E 02 3.0437E 02 3.0426E 02 3.0418E 02 3.0412E 02
         3.0408E 02 3.0405E 02 3.0402E 02 3.0400E 02 3.0400E 02 3.0400E 02 3.0400E 02 3.0400E 02 3.0400E 02 3.0402E 02

RHO      8.4503E-01 8.4545E-01 8.4589E-01 8.4640E-01 8.4690E-01 8.4736E-01 8.4774E-01 8.4803E-01 8.4826E-01 8.4842E-01
         8.4854E-01 8.4863E-01 8.4870E-01 8.4875E-01 8.4878E-01 8.4877E-01 8.4877E-01 8.4877E-01 8.4877E-01 8.4872E-01

H        3.0641E 05 3.0626E 05 3.0610E 05 3.0592E 05 3.0573E 05 3.0557E 05 3.0543E 05 3.0533E 05 3.0525E 05 3.0519E 05
         3.0515E 05 3.0511E 05 3.0509E 05 3.0507E 05 3.0506E 05 3.0506E 05 3.0506E 05 3.0506E 05 3.0506E 05 3.0508E 05

**DIST. ALONG X= 0.1200****NEAR WALL RE=1.0854E 03

Z        3.0085E-04 5.9649E-04 9.4549E-04 1.4782E-03 2.2235E-03 3.2072E-03 4.4530E-03 5.9833E-03 7.8197E-03 9.9840E-03
         1.2499E-02 1.5391E-02 1.8667E-02 2.2309E-02 2.6333E-02 3.0780E-02 3.5720E-02 4.1153E-02 4.7105E-02 5.3597E-02

U        7.8898E 01 8.8637E 01 9.5890E 01 1.0355E 02 1.1121E 02 1.1872E 02 1.2602E 02 1.3306E 02 1.3978E 02 1.4613E 02
         1.5202E 02 1.5731E 02 1.6179E 02 1.6517E 02 1.6716E 02 1.6733E 02 1.6730E 02 1.6731E 02 1.6730E 02 1.6730E 02

T        3.0684E 02 3.0692E 02 3.0692E 02 3.0686E 02 3.0676E 02 3.0665E 02 3.0653E 02 3.0641E 02 3.0631E 02 3.0621E 02
         3.0614E 02 3.0607E 02 3.0602E 02 3.0599E 02 3.0597E 02 3.0596E 02 3.0596E 02 3.0596E 02 3.0596E 02 3.0599E 02

RHO      8.5793E-01 8.5770E-01 8.5771E-01 8.5787E-01 8.5814E-01 8.5846E-01 8.5880E-01 8.5912E-01 8.5942E-01 8.5968E-01
         8.5990E-01 8.6007E-01 8.6021E-01 8.6031E-01 8.6037E-01 8.6039E-01 8.6038E-01 8.6038E-01 8.6038E-01 8.6031E-01

H        3.0791E 05 3.0799E 05 3.0799E 05 3.0793E 05 3.0784E 05 3.0772E 05 3.0760E 05 3.0748E 05 3.0738E 05 3.0729E 05
         3.0721E 05 3.0714E 05 3.0710E 05 3.0706E 05 3.0704E 05 3.0703E 05 3.0704E 05 3.0704E 05 3.0704E 05 3.0706E 05

**DIST. ALONG X= 0.1700****NEAR WALL RE=1.0513E 03

Z        3.1814E-04 6.2990E-04 9.9785E-04 1.5592E-03 2.3439E-03 3.3785E-03 4.6870E-03 6.2918E-03 8.2139E-03 1.0474E-02
         1.3095E-02 1.6099E-02 1.9490E-02 2.3245E-02 2.7373E-02 3.1915E-02 3.6928E-02 4.2449E-02 4.8499E-02 5.5097E-02

U        7.1570E 01 8.0455E 01 8.7108E 01 9.4156E 01 1.0125E 02 1.0824E 02 1.1508E 02 1.2173E 02 1.2814E 02 1.3427E 02
         1.4005E 02 1.4537E 02 1.5005E 02 1.5387E 02 1.5656E 02 1.5781E 02 1.5773E 02 1.5774E 02 1.5774E 02 1.5774E 02

T        3.0787E 02 3.0813E 02 3.0824E 02 3.0829E 02 3.0828E 02 3.0823E 02 3.0815E 02 3.0806E 02 3.0796E 02 3.0786E 02
         3.0777E 02 3.0769E 02 3.0762E 02 3.0756E 02 3.0753E 02 3.0751E 02 3.0751E 02 3.0751E 02 3.0751E 02 3.0754E 02

RHO      8.6830E-01 8.6758E-01 8.6727E-01 8.6713E-01 8.6715E-01 8.6730E-01 8.6751E-01 8.6777E-01 8.6805E-01 8.6832E-01
         8.6858E-01 8.6881E-01 8.6901E-01 8.6917E-01 8.6927E-01 8.6933E-01 8.6933E-01 8.6932E-01 8.6932E-01 8.6924E-01

H        3.0895E 05 3.0921E 05 3.0932E 05 3.0937E 05 3.0936E 05 3.0931E 05 3.0923E 05 3.0914E 05 3.0904E 05 3.0894E 05
         3.0885E 05 3.0877E 05 3.0870E 05 3.0864E 05 3.0860E 05 3.0858E 05 3.0858E 05 3.0858E 05 3.0858E 05 3.0861E 05

**DIST. ALONG X= 0.2200****NEAR WALL RE=1.0211E 03

Z        3.3498E-04 6.6236E-04 1.0486E-03 1.6377E-03 2.4606E-03 3.5448E-03 4.9146E-03 6.5924E-03 8.5990E-03 1.0955E-02
         1.3680E-02 1.6797E-02 2.0305E-02 2.4178E-02 2.8417E-02 3.3057E-02 3.8153E-02 4.3756E-02 4.9897E-02 5.6596E-02

U        6.5502E 01 7.3685E 01 7.9825E 01 8.6352E 01 9.2941E 01 9.9472E 01 1.0589E 02 1.1217E 02 1.1828E 02 1.2418E 02
         1.2981E 02 1.3508E 02 1.3985E 02 1.4393E 02 1.4711E 02 1.4918E 02 1.4966E 02 1.4959E 02 1.4960E 02 1.4959E 02

T        3.0863E 02 3.0902E 02 3.0922E 02 3.0936E 02 3.0942E 02 3.0943E 02 3.0940E 02 3.0934E 02 3.0926E 02 3.0918E 02
         3.0909E 02 3.0900E 02 3.0892E 02 3.0885E 02 3.0880E 02 3.0876E 02 3.0875E 02 3.0875E 02 3.0875E 02 3.0879E 02

RHO      8.7670E-01 8.7559E-01 8.7502E-01 8.7464E-01 8.7446E-01 8.7443E-01 8.7452E-01 8.7469E-01 8.7490E-01 8.7514E-01
         8.7539E-01 8.7564E-01 8.7587E-01 8.7607E-01 8.7623E-01 8.7633E-01 8.7637E-01 8.7635E-01 8.7635E-01 8.7626E-01

H        3.0971E 05 3.1010E 05 3.1030E 05 3.1044E 05 3.1050E 05 3.1051E 05 3.1048E 05 3.1042E 05 3.1035E 05 3.1026E 05
         3.1017E 05 3.1008E 05 3.1000E 05 3.0993E 05 3.0988E 05 3.0984E 05 3.0983E 05 3.0983E 05 3.0983E 05 3.0987E 05
```

DIST. ALONG X= 0.2700**NEAR WALL RE=9.8469E 02

Z	3.6020E-04	7.1050E-04	1.1225E-03	1.7487E-03	2.6204E-03	3.7648E-03	5.2060E-03	6.9658E-03	9.0643E-03	1.1521E-02
	1.4354E-02	1.7585E-02	2.1209E-02	2.5196E-02	2.9543E-02	3.4280E-02	3.9452E-02	4.5120E-02	5.1330E-02	5.8105E-02
U	5.8322E 01	6.5847E 01	7.1575E 01	7.7720E 01	8.3965E 01	9.0184E 01	9.6325E 01	1.0236E 02	1.0826E 02	1.1400E 02
	1.1952E 02	1.2477E 02	1.2962E 02	1.3388E 02	1.3741E 02	1.4000E 02	1.4138E 02	1.4137E 02	1.4137E 02	1.4136E 02
T	3.0939E 02	3.0989E 02	3.1016E 02	3.1036E 02	3.1048E 02	3.1054E 02	3.1055E 02	3.1053E 02	3.1048E 02	3.1041E 02
	3.1033E 02	3.1025E 02	3.1016E 02	3.1009E 02	3.1002E 02	3.0997E 02	3.0994E 02	3.0994E 02	3.0994E 02	3.0998E 02
RHO	8.8454E-01	8.8311E-01	8.8234E-01	8.8177E-01	8.8142E-01	8.8125E-01	8.8122E-01	8.8129E-01	8.8143E-01	8.8162E-01
	8.8184E-01	8.8208E-01	8.8232E-01	8.8254E-01	8.8273E-01	8.8287E-01	8.8296E-01	8.8296E-01	8.8296E-01	8.8284E-01
H	3.1047E 05	3.1097E 05	3.1124E 05	3.1144E 05	3.1157E 05	3.1163E 05	3.1164E 05	3.1161E 05	3.1156E 05	3.1150E 05
	3.1142E 05	3.1133E 05	3.1125E 05	3.1117E 05	3.1111E 05	3.1105E 05	3.1102E 05	3.1102E 05	3.1102E 05	3.1107E 05

DIST. ALONG X= 0.3200**NEAR WALL RE=9.5409E 02

Z	3.8037E-04	7.4906E-04	1.1824E-03	1.8406E-03	2.7559E-03	3.9559E-03	5.4648E-03	7.3036E-03	9.4917E-03	1.2047E-02
	1.4989E-02	1.8334E-02	2.2076E-02	2.6181E-02	3.0640E-02	3.5482E-02	4.0742E-02	4.6479E-02	5.2754E-02	5.9601E-02
U	5.3194E 01	6.0131E 01	6.5406E 01	7.1092E 01	7.6908E 01	8.2742E 01	8.8545E 01	9.4289E 01	9.9947E 01	1.0549E 02
	1.1088E 02	1.1605E 02	1.2090E 02	1.2527E 02	1.2904E 02	1.3203E 02	1.3403E 02	1.3459E 02	1.3451E 02	1.3452E 02
T	3.0991E 02	3.1050E 02	3.1084E 02	3.1110E 02	3.1128E 02	3.1138E 02	3.1143E 02	3.1144E 02	3.1141E 02	3.1137E 02
	3.1130E 02	3.1123E 02	3.1115E 02	3.1107E 02	3.1100E 02	3.1094E 02	3.1089E 02	3.1087E 02	3.1088E 02	3.1092E 02
RHO	8.9089E-01	8.8919E-01	8.8823E-01	8.8749E-01	8.8693E-01	8.8669E-01	8.8655E-01	8.8652E-01	8.8659E-01	8.8673E-01
	8.8691E-01	8.8712E-01	8.8735E-01	8.8757E-01	8.8778E-01	8.8796E-01	8.8809E-01	8.8814E-01	8.8813E-01	8.8799E-01
H	3.1100E 05	3.1159E 05	3.1193E 05	3.1219E 05	3.1237E 05	3.1247E 05	3.1252E 05	3.1253E 05	3.1250E 05	3.1246E 05
	3.1239E 05	3.1232E 05	3.1224E 05	3.1216E 05	3.1208E 05	3.1202E 05	3.1198E 05	3.1196E 05	3.1196E 05	3.1201E 05

DIST. ALONG X= 0.3700**NEAR WALL RE=9.2340E 02

Z	4.0496E-04	7.9573E-04	1.2539E-03	1.9483E-03	2.9115E-03	4.1712E-03	5.7510E-03	7.6717E-03	9.9518E-03	1.2608E-02
	1.5658E-02	1.9118E-02	2.2977E-02	2.7198E-02	3.1770E-02	3.6715E-02	4.2066E-02	4.7872E-02	5.4200E-02	6.1103E-02
U	4.8116E 01	5.4550E 01	5.9470E 01	6.4804E 01	7.0294E 01	7.5827E 01	8.1355E 01	8.6850E 01	9.2292E 01	9.7655E 01
	1.0291E 02	1.0800E 02	1.1284E 02	1.1727E 02	1.2120E 02	1.2448E 02	1.2692E 02	1.2824E 02	1.2824E 02	1.2823E 02
T	3.1039E 02	3.1105E 02	3.1143E 02	3.1174E 02	3.1195E 02	3.1209E 02	3.1218E 02	3.1221E 02	3.1221E 02	3.1219E 02
	3.1214E 02	3.1208E 02	3.1200E 02	3.1193E 02	3.1185E 02	3.1179E 02	3.1173E 02	3.1170E 02	3.1169E 02	3.1175E 02
RHO	8.9633E-01	8.9443E-01	8.9334E-01	8.9247E-01	8.9185E-01	8.9145E-01	8.9121E-01	8.9111E-01	8.9111E-01	8.9118E-01
	8.9132E-01	8.9149E-01	8.9170E-01	8.9192E-01	8.9213E-01	8.9233E-01	8.9248E-01	8.9259E-01	8.9259E-01	8.9244E-01
H	3.1148E 05	3.1214E 05	3.1252E 05	3.1283E 05	3.1304E 05	3.1319E 05	3.1327E 05	3.1330E 05	3.1331E 05	3.1328E 05
	3.1323E 05	3.1317E 05	3.1310E 05	3.1302E 05	3.1295E 05	3.1288E 05	3.1282E 05	3.1279E 05	3.1279E 05	3.1284E 05

DIST. ALONG X= 0.4200**NEAR WALL RE= 9.9785E 02

Z	4.2430E-04	8.3274E-04	1.3116E-03	2.0369E-03	3.0423E-03	4.3559E-03	6.0015E-03	7.9994E-03	1.0367E-02	1.3122E-02
	1.6278E-02	1.9851E-02	2.3829E-02	2.8168E-02	3.2856E-02	3.7911E-02	4.3362E-02	4.9250E-02	5.5639E-02	6.2597E-02
U	4.4466E 01	5.0411E 01	5.4989E 01	5.9965E 01	6.5104E 01	7.0309E 01	7.5538E 01	8.0765E 01	8.5970E 01	9.1132E 01
	9.6222E 01	1.0120E 02	1.0597E 02	1.1042E 02	1.1444E 02	1.1792E 02	1.2071E 02	1.2256E 02	1.2304E 02	1.2295E 02
T	3.1074E 02	3.1146E 02	3.1188E 02	3.1222E 02	3.1247E 02	3.1264E 02	3.1275E 02	3.1281E 02	3.1284E 02	3.1283E 02
	3.1280E 02	3.1275E 02	3.1269E 02	3.1261E 02	3.1254E 02	3.1247E 02	3.1241E 02	3.1237E 02	3.1235E 02	3.1241E 02

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RHO       9.0077E-01 8.9869E-01 8.9748E-01 8.9649E-01 8.9577E-01 8.9528E-01 8.9497E-01 8.9480E-01 8.9473E-01 8.9476E-01
          8.9484E-01 8.9499E-01 8.9517E-01 8.9537E-01 8.9558E-01 8.9577E-01 8.9595E-01 8.9608E-01 8.9614E-01 8.9597E-01

H         3.1183E 05 3.1255E 05 3.1297E 05 3.1332E 05 3.1357E 05 3.1374E 05 3.1385E 05 3.1391E 05 3.1393E 05 3.1392E 05
          3.1389E 05 3.1384E 05 3.1378E 05 3.1371E 05 3.1364E 05 3.1357E 05 3.1351E 05 3.1346E 05 3.1344E 05 3.1350E 05

**DIST. ALONG X= 0.4700****NEAR WALL RE=8.7183E 02

Z         4.4957E-04 8.8055E-04 1.3846E-03 2.1462E-03 3.1993E-03 4.5719E-03 6.2875E-03 8.3659E-03 1.0824E-02 1.3678E-02
          1.6940E-02 2.0627E-02 2.4721E-02 2.9176E-02 3.3977E-02 3.9139E-02 4.4687E-02 5.0657E-02 5.7104E-02 6.4103E-02

U         4.0609E 01 4.6162E 01 5.0473E 01 5.5188E 01 6.0072E 01 6.5032E 01 7.0026E 01 7.5035E 01 8.0041E 01 8.5029E 01
          8.9973E 01 9.4841E 01 9.9552E 01 1.0399E 02 1.0807E 02 1.1169E 02 1.1473E 02 1.1696E 02 1.1810E 02 1.1905E 02

T         3.1108E 02 3.1184E 02 3.1229E 02 3.1265E 02 3.1293E 02 3.1313E 02 3.1326E 02 3.1334E 02 3.1339E 02 3.1339E 02
          3.1338E 02 3.1334E 02 3.1329E 02 3.1322E 02 3.1316E 02 3.1309E 02 3.1302E 02 3.1297E 02 3.1294E 02 3.1299E 02

RHO       9.0460E-01 9.0241E-01 9.0112E-01 9.0006E-01 8.9926E-01 8.9870E-01 8.9832E-01 8.9809E-01 8.9797E-01 8.9794E-01
          8.9798E-01 8.9809E-01 8.9824E-01 8.9842E-01 8.9861E-01 8.9881E-01 8.9899E-01 8.9915E-01 8.9925E-01 8.9909E-01

H         3.1217E 05 3.1293E 05 3.1338E 05 3.1375E 05 3.1403E 05 3.1422E 05 3.1436E 05 3.1444E 05 3.1448E 05 3.1449E 05
          3.1447E 05 3.1444E 05 3.1438E 05 3.1432E 05 3.1425E 05 3.1418E 05 3.1412E 05 3.1407E 05 3.1403E 05 3.1409E 05

**DIST. ALONG X= 0.5200****NEAR WALL RE=8.4911E 02

Z         4.7024E-04 9.1974E-04 1.4452E-03 2.2387E-03 3.3350E-03 4.7624E-03 6.5441E-03 8.6994E-03 1.1245E-02 1.4195E-02
          1.7562E-02 2.1359E-02 2.5569E-02 3.0141E-02 3.5056E-02 4.0328E-02 4.5978E-02 5.2038E-02 5.8552E-02 6.5596E-02

U         3.7700E 01 4.2893E 01 4.6922E 01 5.1341E 01 5.5942E 01 6.0638E 01 6.5389E 01 7.0175E 01 7.4979E 01 7.9786E 01
          8.4577E 01 8.9321E 01 9.3947E 01 9.8348E 01 1.0244E 02 1.0616E 02 1.0937E 02 1.1192E 02 1.1357E 02 1.1390E 02

T         3.1134E 02 3.1214E 02 3.1261E 02 3.1301E 02 3.1331E 02 3.1352E 02 3.1367E 02 3.1377E 02 3.1382E 02 3.1385E 02
          3.1385E 02 3.1382E 02 3.1378E 02 3.1373E 02 3.1367E 02 3.1360E 02 3.1354E 02 3.1348E 02 3.1344E 02 3.1348E 02

RHO       9.0784E-01 9.0552E-01 9.0415E-01 9.0301E-01 9.0216E-01 9.0154E-01 9.0111E-01 9.0083E-01 9.0066E-01 9.0059E-01
          9.0059E-01 9.0066E-01 9.0078E-01 9.0094E-01 9.0112E-01 9.0130E-01 9.0149E-01 9.0165E-01 9.0178E-01 9.0167E-01

H         3.1243E 05 3.1323E 05 3.1371E 05 3.1410E 05 3.1440E 05 3.1462E 05 3.1477E 05 3.1487E 05 3.1492E 05 3.1495E 05
          3.1495E 05 3.1492E 05 3.1488E 05 3.1483E 05 3.1476E 05 3.1470E 05 3.1464E 05 3.1458E 05 3.1453E 05 3.1457E 05
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EX	PRESS	P-RECV	WALL SHEAR (Pa)	HEAT FLX (W/m²)	B.L. THK	DISP THK	MOM THK	SHAPE FAC
0.0	7.100E 04	0.0	3.967E 01	0.0	1.553E-02	1.913E-03	1.471E-03	1.301E 00
3.906E-05	7.100E 04	1.536E-04	3.963E 01	7.372E 00	1.554E-02	1.913E-03	1.471E-03	1.301E 00
7.812E-05	7.100E 04	2.583E-04	3.961E 01	1.378E 01	1.554E-02	1.914E-03	1.471E-03	1.301E 00
1.562E-04	7.101E 04	5.340E-04	3.955E 01	2.754E 01	1.555E-02	1.915E-03	1.472E-03	1.301E 00
3.125E-04	7.102E 04	1.054E-03	3.945E 01	5.365E 01	1.556E-02	1.917E-03	1.473E-03	1.301E 00
6.250E-04	7.104E 04	2.084E-03	3.925E 01	1.027E 02	1.559E-02	1.921E-03	1.476E-03	1.301E 00
1.250E-03	7.107E 04	4.119E-03	3.890E 01	1.899E 02	1.564E-02	1.929E-03	1.481E-03	1.302E 00
2.500E-03	7.115E 04	8.195E-03	3.829E 01	3.364E 02	1.574E-02	1.945E-03	1.492E-03	1.304E 00
5.000E-03	7.129E 04	1.624E-02	3.727E 01	5.637E 02	1.592E-02	1.978E-03	1.513E-03	1.308E 00
1.000E-02	7.157E 04	3.189E-02	3.565E 01	8.906E 02	1.625E-02	2.043E-03	1.555E-03	1.314E 00
2.000E-02	7.211E 04	6.176E-02	3.306E 01	1.330E 03	1.690E-02	2.172E-03	1.641E-03	1.324E 00
3.000E-02	7.252E 04	8.508E-02	3.159E 01	1.596E 03	1.814E-02	2.282E-03	1.717E-03	1.329E 00
4.000E-02	7.294E 04	1.085E-01	2.994E 01	1.822E 03	1.902E-02	2.401E-03	1.798E-03	1.335E 00
5.000E-02	7.333E 04	1.301E-01	2.862E 01	1.997E 03	1.970E-02	2.517E-03	1.878E-03	1.340E 00
6.000E-02	7.371E 04	1.512E-01	2.727E 01	2.149E 03	2.032E-02	2.639E-03	1.963E-03	1.344E 00
7.000E-02	7.405E 04	1.705E-01	2.616E 01	2.272E 03	2.092E-02	2.759E-03	2.047E-03	1.348E 00
8.000E-02	7.438E 04	1.889E-01	2.509E 01	2.377E 03	2.150E-02	2.881E-03	2.131E-03	1.352E 00
9.000E-02	7.470E 04	2.067E-01	2.407E 01	2.468E 03	2.265E-02	3.007E-03	2.219E-03	1.355E 00
1.000E-01	7.500E 04	2.234E-01	2.313E 01	2.546E 03	2.354E-02	3.135E-03	2.307E-03	1.359E 00
1.100E-01	7.528E 04	2.391E-01	2.229E 01	2.612E 03	2.427E-02	3.262E-03	2.396E-03	1.362E 00
1.200E-01	7.555E 04	2.542E-01	2.148E 01	2.668E 03	2.493E-02	3.392E-03	2.486E-03	1.365E 00
1.300E-01	7.581E 04	2.686E-01	2.071E 01	2.717E 03	2.556E-02	3.524E-03	2.577E-03	1.367E 00
1.400E-01	7.606E 04	2.823E-01	1.997E 01	2.758E 03	2.617E-02	3.660E-03	2.671E-03	1.370E 00
1.500E-01	7.629E 04	2.952E-01	1.931E 01	2.792E 03	2.678E-02	3.794E-03	2.764E-03	1.373E 00
1.600E-01	7.651E 04	3.076E-01	1.867E 01	2.821E 03	2.776E-02	3.930E-03	2.858E-03	1.375E 00
1.700E-01	7.672E 04	3.196E-01	1.805E 01	2.845E 03	2.878E-02	4.071E-03	2.954E-03	1.378E 00
1.800E-01	7.693E 04	3.309E-01	1.746E 01	2.865E 03	2.962E-02	4.213E-03	3.052E-03	1.380E 00
1.900E-01	7.712E 04	3.416E-01	1.693E 01	2.881E 03	3.034E-02	4.354E-03	3.149E-03	1.383E 00
2.000E-01	7.730E 04	3.520E-01	1.641E 01	2.893E 03	3.101E-02	4.497E-03	3.247E-03	1.385E 00
2.100E-01	7.748E 04	3.619E-01	1.591E 01	2.902E 03	3.166E-02	4.644E-03	3.347E-03	1.388E 00
2.200E-01	7.766E 04	3.717E-01	1.539E 01	2.907E 03	3.230E-02	4.796E-03	3.449E-03	1.390E 00
2.300E-01	7.784E 04	3.822E-01	1.472E 01	2.904E 03	3.295E-02	4.969E-03	3.562E-03	1.395E 00
2.400E-01	7.801E 04	3.917E-01	1.421E 01	2.903E 03	3.370E-02	5.137E-03	3.673E-03	1.399E 00
2.500E-01	7.819E 04	4.014E-01	1.362E 01	2.895E 03	3.491E-02	5.318E-03	3.789E-03	1.404E 00
2.600E-01	7.838E 04	4.122E-01	1.287E 01	2.876E 03	3.590E-02	5.528E-03	3.918E-03	1.411E 00
2.700E-01	7.854E 04	4.212E-01	1.244E 01	2.869E 03	3.674E-02	5.716E-03	4.038E-03	1.415E 00
2.800E-01	7.870E 04	4.298E-01	1.198E 01	2.858E 03	3.750E-02	5.905E-03	4.159E-03	1.420E 00
2.900E-01	7.884E 04	4.378E-01	1.161E 01	2.849E 03	3.821E-02	6.087E-03	4.277E-03	1.423E 00
3.000E-01	7.898E 04	4.456E-01	1.122E 01	2.836E 03	3.890E-02	6.276E-03	4.398E-03	1.427E 00
3.100E-01	7.911E 04	4.530E-01	1.086E 01	2.823E 03	3.958E-02	6.465E-03	4.519E-03	1.430E 00
3.200E-01	7.924E 04	4.602E-01	1.051E 01	2.808E 03	4.026E-02	6.657E-03	4.642E-03	1.434E 00
3.300E-01	7.936E 04	4.670E-01	1.019E 01	2.794E 03	4.093E-02	6.849E-03	4.765E-03	1.437E 00
3.400E-01	7.948E 04	4.735E-01	9.890E 00	2.780E 03	4.214E-02	7.042E-03	4.887E-03	1.441E 00
3.500E-01	7.962E 04	4.813E-01	9.334E 00	2.741E 03	4.326E-02	7.288E-03	5.031E-03	1.449E 00
3.600E-01	7.974E 04	4.879E-01	9.042E 00	2.726E 03	4.419E-02	7.504E-03	5.164E-03	1.453E 00
3.700E-01	7.985E 04	4.941E-01	8.735E 00	2.705E 03	4.500E-02	7.718E-03	5.295E-03	1.458E 00
3.800E-01	7.995E 04	4.999E-01	8.489E 00	2.690E 03	4.575E-02	7.925E-03	5.424E-03	1.461E 00
3.900E-01	8.005E 04	5.056E-01	8.234E 00	2.671E 03	4.646E-02	8.135E-03	5.553E-03	1.465E 00
4.000E-01	8.015E 04	5.110E-01	8.002E 00	2.653E 03	4.716E-02	8.345E-03	5.683E-03	1.468E 00
4.100E-01	8.024E 04	5.162E-01	7.775E 00	2.635E 03	4.785E-02	8.556E-03	5.813E-03	1.472E 00
4.200E-01	8.033E 04	5.212E-01	7.557E 00	2.616E 03	4.854E-02	8.770E-03	5.944E-03	1.475E 00
4.300E-01	8.042E 04	5.260E-01	7.347E 00	2.598E 03	4.922E-02	8.986E-03	6.076E-03	1.479E 00
4.400E-01	8.050E 04	5.307E-01	7.149E 00	2.579E 03	5.010E-02	9.204E-03	6.209E-03	1.482E 00
4.500E-01	8.059E 04	5.356E-01	6.886E 00	2.551E 03	5.141E-02	9.446E-03	6.351E-03	1.487E 00
4.600E-01	8.068E 04	5.408E-01	6.562E 00	2.515E 03	5.251E-02	9.720E-03	6.505E-03	1.494E 00
4.700E-01	8.076E 04	5.453E-01	6.382E 00	2.497E 03	5.348E-02	9.966E-03	6.650E-03	1.499E 00
4.800E-01	8.084E 04	5.496E-01	6.183E 00	2.474E 03	5.437E-02	1.022E-02	6.797E-03	1.503E 00
4.900E-01	8.091E 04	5.537E-01	6.029E 00	2.457E 03	5.521E-02	1.046E-02	6.942E-03	1.506E 00
5.000E-01	8.099E 04	5.577E-01	5.858E 00	2.436E 03	5.604E-02	1.071E-02	7.090E-03	1.510E 00
5.100E-01	8.105E 04	5.615E-01	5.712E 00	2.418E 03	5.684E-02	1.096E-02	7.238E-03	1.514E 00

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APPENDIX A

In the computation of confined flows the pressure gradient along the flow is usually the unknown parameter to be calculated. The finite difference equations given in section 2, along with the supporting relations given in Appendix A of Ref. 7, favor the case in which the pressure gradient is the specified varaible and the duct area along the flow is the unknown quantity to be calculated. When the pressure gradient is the unknown quantity these equations, as discussed earlier, are solved iteratively. Two questions arise in regard to the pressure iteration. One, how to adjust the previously used Δp (pressure drop across Δx) for the succeeding iteration. Second, how best to estimate starting Δp at the beginning of a new integration step. In this appendix we have summarized our current know-how in regard to these two questions.

Applying conservation principles approximately across x and $x+\Delta x$ we can derive the following equations:

$$p' = \frac{\dot{m}U_2}{A_1A_2\left(1 - \frac{\dot{m}U_2}{A_1p_2}\right)} A' \tag{A-1}$$

$$U' = \frac{U_2}{A_2\left(\frac{\dot{m}U_2}{A_1p_2} - 1\right)} A' = -\frac{A_1}{\dot{m}}p' \tag{A-2}$$

Equation (A-1) is derived in Appendix C of Ref. 7 and Eq. (A-2) follows easily from the equations given in that appendix. In these equations

U_2 is average velocity at $x + \Delta x$

A_1 duct area at x,

A_2 duct area at $x + \Delta x$

$\dot{m}$ mass flow rate

p_2 pressure at $x + \Delta x$

A' small change in A_2

p' small change in Δp

U' small change in U_2

θ = infinity for incompressible flows

= ratio of specific heats for compressible flows

We also introduce the following nomenclature for later use:

DPDTL specified tolerance within which the pressure gradient is to be calculated (put into the program as a dimensionless number, DPDTL = 0.01 means that the pressure graduent is to be calculated within ±1%),

ZTLRN tolerance for the calculation of the duct area,

UTLRN tolerance in the calculation of velocity at $x + \Delta x$.

At the beginning of the MAIN program DPDTL is specified through an assignment statement. Given DPDTL (p') Eq. (A-1) specifies the tolerance limit within which the duct area must be calculated to meet the pressure gradient tolerance. Equation (A-2) tells how closely U_2 must be calculated during the velocity iteration loop. At the beginning of a new integration step Eqs. (A-1) and (A-2) are used to calculate ZTLRN (A') and UTLRN (U') from the specified DPDTL (p'). Then with an initial estimate of Δp we start the velocity iteration loop until U_2 at $x + \Delta x$ converges within

UTLRN. We next check calculated A_2 against the specified A_2. If the difference, called A', is greater than ZTLRN we change Δp by p', calculated from Eq. (A-1) using A', for the succeeding pressure iteration. Note Eq. (A-1) does double duty--at the beginning of a new integration step it is used to calculate ZTLRN for that integration step and then during the pressure iteration loop it is used to correct Δp for the next iteration using A' from the last iteration. In the actual computations Eqs. (A-1) and (A-2) are used with U_2 replaced by the known centerline velocity at x, p_2 by p_1, A_2 by A_1, and U' as the variation of U_2 (U along streamline 2). To make up partly for these approximations and partly for those that went into the derivation of Eq. (A-1), Eq. (A-1) when employed within the pressure iteration loop, to calculate adjustment to Δp from A', is used along with a damping factor. Correction p' calculated from (A-1) is decreased by the factor

$$\text{DPDAMP} = 0.5 \times \exp\left(-0.115 \frac{\text{distance along the duct}}{\text{half duct width or radius}}\right) . \qquad \text{(A-3)}$$

We next turn our attention to the estimation of initial Δp, call it ΔpI, at the beginning of each new integration step. Denote the final calculated Δp at the given integration step by ΔpF. In our earlier computations ΔpI at x_i was calculated from ΔpF at x_{i-2} and x_{i-1} using straight line projection. In Fig. (A-1a) is shown the distribution, using this approach, of $\Delta pI(x)$ and $\Delta pF(\odot)$ along part of a duct. The flow is through a rectangular diffuser. Note the flip-flop nature of the distribution of ΔpI with respect to mean $\Delta p/\Delta x$ versus x curve. Note

× Initial estimates of $\Delta p/\Delta X$

⊙ Final calculated values of $\Delta p/\Delta X$

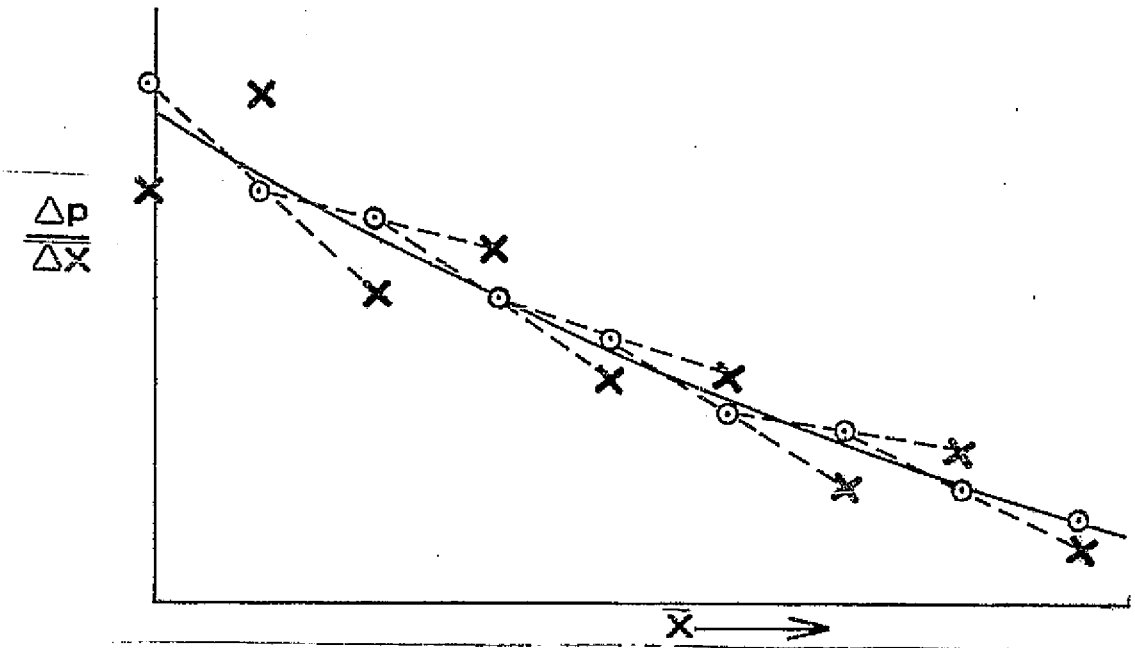

Fig. (A-1a). Initial pressure gradient estimated from previous two calculated values using a linear relation.

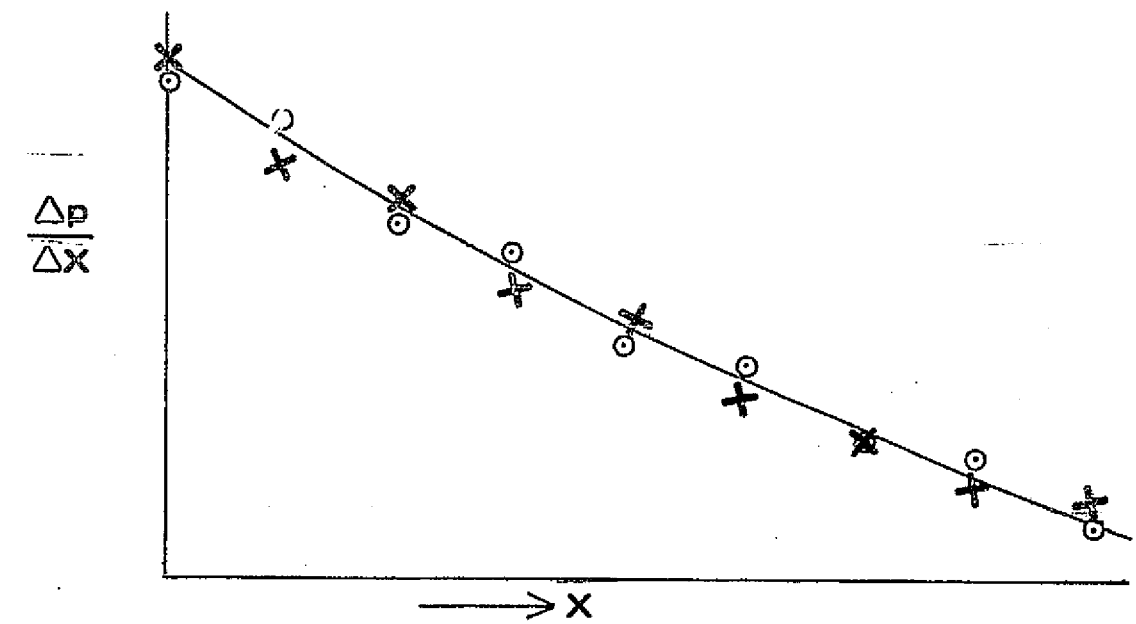

Fig. (A-1b). Initial pressure gradient estimated from previous three calculated values using regression analysis.

also that to estimate ΔpI by fitting second order curve through ΔpF at x_{i-3}, x_{i-2}, and x_{i-1} will worsen the estimate of ΔpI.

Since ΔpF is calculated not exactly but within a specified limit, the values of ΔpF will fall randomly about the mean $\Delta p/\Delta x$ versus x curve. Thus ΔpI at x_i should be calculated from ΔpF at previous steps on statistical basis. In our later computations ΔpI at x_i was projected from ΔpF at x_{i-3}, x_{i-2}, and x_{i-1} using regression analysis. Distribution of ΔpI and ΔpF using this approach is shown in Fig. (A-1b). Figures (A-1a) and (A-1b) are for the same flow over the same section of the duct. Estimation of ΔpI using regression analysis has improved the convergence rate in our computations.

In the program listing given in section 3 ΔpI at x_i is calculated from previous three values of ΔpF using the following formulae:

$$\Delta p = \text{DPSLOP}\ x_i + \text{DPZERO} \tag{A-4}$$

$$\text{DPSLOP} = \frac{3 \Sigma \Delta p_i x_i - (\Sigma \Delta p_i)(\Sigma x_i)}{3 \Sigma x_i^2 - (\Sigma x_i)^2} \tag{A-5}$$

$$\text{DPZERO} = \frac{(\Sigma \Delta p_i)(\Sigma x_i^2) - (\Sigma x_i)(\Sigma x_i \Delta p_i)}{3 \Sigma x_i^2 - (\Sigma x_i)^2} \tag{A-6}$$

In the program x at the first of the three points to be projected is taken equal to zero. Near the entrance before three ΔpF are available ΔpI is set simply equal to the last ΔpF.

APPENDIX B

STREAMWISE COMPUTATION OF DUCT FLOWS

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ABSTRACT

A computational method to calculate momentum and energy transport in two-dimensional viscous compressible duct flows is presented. The flow in the duct is partitioned into finite streams. The difference equations are then obtained by applying momentum and energy conservation principles directly to the individual streams. The method is applicable to laminar and turbulent flows.

I. INTRODUCTION

Two-dimensional effects in duct flows, such as the growth of momentum and thermal boundary layers, have been studied for a long time by separating the flow into a core flow and a boundary layer [1]. Two-dimensional effects associated with sudden changes in duct shape in incompressible flows have been successfully computed using vorticity transport equation [2]. In high temperature compressible flows such as those encountered in magnetohydrodynamic (MHD) channels separation of flow into a uniform core and a boundary layer is difficult except at the very beginning of the channel. The difficulties arise from (1) large temperature differences between the wall and the core flow which lead to temperature gradients within the core and (2) shrinking size of the core as the flow proceeds down the channel. Also in calculating performance of diffusers with high inlet blockage factor, as in MHD diffusers, one cannot separate the flow into a core and a boundary layer. Computation of such flows have been recently carried out mostly using equations obtained by von Mises transformation of boundary layer equations [3,4,5]. Some MHD channel calculations have also been carried out using Fluid-in-Cell Method [6,7].

We have in this article presented a new approach to compute internal flows without separating the flow into a core and a boundary layer. A novel feature of this approach is the method by which the difference equations are obtained. In most of the approaches one starts with the differential equations, such as energy equation, Navier-Stokes equations, which are obtained form the conservation principles. These equations

either in their primitive forms or in transformed forms are then differenced to obtain the finite difference equations needed to compute the flow. In the present approach the flow in the duct is portioned into finite streams and the finite difference equations are then obtained by applying conservation principles directly to the individual streams. The resulting equations are similar to the ones obtained previously [3,4] by the procedure: derive boundary layer equations from the conservation principles via energy and Navier-Stokes equations--carry out von Mises transformation of the boundary layer equations--finite difference the transformed equations using integral method finite differencing. In contrast, in the present approach there is direct relationship between the finite difference equations and their roots, the conservation principles. This direct relationship, we think, makes it easier to introduce approximations allowed by the flow being simulated and also to incorporate any special features of the flow.

In section II we have presented the finite difference equations based on the new approach and in section III discussed their solution techniques. The discussion is limited to two-dimensional flows. The application of the present approach to compute three-dimensional effects is straightforward and is indicated at the end of section III. In the main body of the paper we have tried to present the new approach in broad and general terms to make its application easier to specific problems requiring individual handling. Some specific details are given in Appendices A and B. To provide an assessment of the present approach we have carried out sample computations of turbulent flow in a smooth pipe. The results are shown in Appendix C and are compared

with experimental data and other works. We have also used this approach to compute two-dimensional viscous compressible turbulent flows in hydrogen-oxygen combustion driven MHD ducts [8] and are currently using it to calculate diffuser performance with high inlet blockage factors.

II. FINITE DIFFERENCE EQUATIONS

Consider two dimensional duct flow as shown in Fig. 1. The direction x is taken along the flow and z, measured from the duct wall, normal to it. For circular ducts z represents the radial distance from the wall. For rectangular ducts the variation of flow properties normal to z and x are neglected. We partition the total flow in the duct into a sequence of finite streams. To accomplish this partitioning we draw a series of streamlines $1,2,\ldots,N$ as shown in Fig. 1. Streamline 1 is taken along the wall and streamline N represents the duct center or the symmetry line. The spacing between any two consecutive streamlines is determined by the desired resolution of the local gradients normal to the flow. We define streamline $j+1/2$ as the streamline passing through the middle of j and $j+1$. Streamline $j-1/2$ is defined similarly. The stream j, in general, is now defined to be the flow bounded by streamlines $j-1/2$ and $j+1/2$. The shape of the stream, of course, depends on the shape of the duct: in rectangular ducts the streams will be plane layers and in circular ducts the streams will be cylindrical shells. Stream 2 is defined to be the flow between the wall and streamline $2+1/2$. Stream N is defined to be the flow between the streamline $N-1/2$ and the center or symmetry line N.

We introduce the following nomenclature:

ψ_j^+; mass flow rate between the streamline j and $j+1/2$.

ψ_j^-; mass flow rate between the streamline j and $j-1/2$.

Note $\psi_j^+ + \psi_j^-$ represents the mass flow rate through stream j. For calculations of ψ_j's see Appendix A.

$u_j(x)$, $T_j(x)$, $\rho_j(x)$, $h_j(x)$, $C_j(x)$; velocity, temperature, density, enthalpy and constant pressure specific heat along the streamline j.

$z_j(x)$; distance of streamline j from the wall.

A_j^x; cross-sectional area of stream j (see Appendix A).

$\mu_{j+1/2}(x)$, $k_{j+1/2}(x)$; effective viscosity and thermal conductivity (including turbulent contribution when the flow is turbulent) along streamline $j+1/2$. These are the coefficients that are later used to calculate momentum and energy transfer between the stream j and $j+1$ across the interface at $j+1/2$.

Other nomenclature is introduced as we go along.

The difference equations to calculate the flow properties are now obtained by applying conservation principles directly to the individual streams. We assume that the streamlines are chosen sufficiently close so that the variation of the flow properties between any two successive streamlines j and $j+1$ can be approximated as linear. Note the flow properties u_j, T_j, etc., though not their gradients, are continuous across these streamlines. However, across the interface of the streams, i.e. across streamlines $j+1/2$, the gradients are also continuous thus determining, unambiguously, momentum and energy transfer between the adjoining streams.

Applying momentum balance to stream j between x and $x+\Delta x$, i.e. applying the momentum balance to the control volume bounded by x, $x+\Delta x$, $j-1/2$, and $j+1/2$, we obtain,

$$\text{Momentum Flux Out}\Big|_{\text{at } x+\Delta x} - \text{Momentum Flux In}\Big|_{\text{at } x}$$

$$= \text{Pressure Forces} + \text{Viscous Forces} + \text{Body Forces} \qquad (1)$$

Estimation of the various quantities in Eq. (1) allows a great deal of flexibility to accommodate the desired accuracy and any special features of the given flow. We here present one such approximation for stream j, (streams 2 and N require special treatment and are considered later separately).

$$\left[\psi_j^+\left(\frac{3u_j^+ + u_{j+1}^+}{4}\right) + \psi_j^-\left(\frac{3u_j^+ + u_{j-1}^+}{4}\right)\right]$$

$$- \left[\psi_j^+\left(\frac{3u_j + u_{j+1}}{4}\right) + \psi_j^-\left(\frac{3u_j + u_{j-1}}{4}\right)\right]$$

$$= -\Delta p A_j^x + \frac{1}{2}\left[V_{j+1/2}^+(u_{j+1}^+ - u_j^+) + V_{j+1/2}(u_{j+1} - u_j)\right]$$

$$- \frac{1}{2}\left[V_{j-1/2}^+(u_j^+ - u_{j-1}^+) + V_{j-1/2}(u_j - u_{j-1})\right] + F_j \qquad (2)$$

On the left hand side of Eq. (2) the expressions involving ψ_j^+ represents the momentum flux between j and $j+1/2$ and those involving ψ_j^- the momentum flux between $j-1/2$ and j. The superscript + indicates the unknown velocities at $x+\Delta x$, Δp the pressure difference between $x+\Delta x$ and x, and F_j the sum of body forces on the control volume. Also we have introduced

$$V_{j+1/2} = \frac{\mu_{j+1/2}}{z_{j+1} - z_j} A^s_{j+1/2}, \qquad V_{j-1/2} = \frac{\mu_{j-1/2}}{z_j - z_{j-1}} A^s_{j-1/2} \qquad (3)$$

where $A^s_{j+1/2}$ and $A^s_{j-1/2}$ represent the upper and lower surface areas of the control volume (see Appendix A for calculation of A^s's). Terms $V^+_{j+1/2}$ and $V^+_{j-1/2}$ are defined similarly with μ's and z's evaluated at $x+\Delta x$. Note we have estimated the viscous forces on a given surface by taking the average

of those at x and x + Δx. Rewritting Eq. (2) with all the unknown velocities collected on the left hand side we obtain,

$$-u^{+}_{j+1}\left[-\frac{\psi^{+}_{j}}{4}+\frac{1}{2}V^{+}_{j+1/2}\right]+u^{+}_{j}\left[\frac{3}{4}\psi^{+}_{j}+\frac{3}{4}\psi^{-}_{j}+\frac{1}{2}V^{+}_{j+1/2}+\frac{1}{2}V^{+}_{j-1/2}\right]$$

$$-u^{+}_{j-1}\left[-\frac{\psi^{-}_{j}}{4}+\frac{1}{2}V^{+}_{j-1/2}\right]$$

$$=-\Delta pA^{x}_{j}+u_{j+1}\left[\frac{\psi^{+}_{j}}{4}+\frac{1}{2}V_{j+1/2}\right]+u_{j}\left[\frac{3}{4}\psi^{+}_{j}+\frac{3}{4}\psi^{-}_{j}-\frac{1}{2}V_{j+1/2}-\frac{1}{2}V_{j-1/2}\right]$$

$$+u_{j-1}\left[\frac{\psi^{-}_{j}}{4}+\frac{1}{2}V_{j-1/2}\right]+F_{j}$$

$$j=3,4,\ldots,N-1 \qquad (4)$$

Similarly, application of momentum balance to stream N, after some rearrangement, yields

$$u^{+}_{N}\left[\frac{3}{4}\psi^{-}_{N}+\frac{1}{2}V^{+}_{N-1/2}\right]-u^{+}_{N-1}\left[-\frac{\psi^{-}_{N}}{4}+\frac{1}{2}V^{+}_{N-1/2}\right]$$

$$=-\Delta pA^{x}_{N}+u_{N}\left[\frac{3}{4}\psi^{-}_{N}-\frac{1}{2}V_{N-1/2}\right]+u_{N-1}\left[\frac{\psi^{-}_{N}}{4}+\frac{1}{2}V_{N-1/2}\right]+F_{N} \qquad (5)$$

We note that Eq. (4) will yield Eq. (5) for j=N if we were to set ψ^{+}_{N}, $V^{+}_{N+1/2}$, $V_{N+1/2}$, and u_{N+1} equal to zero. Thus by initializing these quantities equal to zero at the beginning of a computer program stream N can be treated by the general formula for stream j.

Next we will treat Stream 2. This stream requires considerably more computational finesse. The sources of the difficulty are (1) occurrence of steep

gradients normal to the flow near the wall and (2) reckoning of wall surface roughness for turbulent flows. For laminar flows the wall surface roughness does not affect the flow and the gradients are not as steep and the flow can be simulated by taking streams sufficiently thin next to the wall. For turbulent flows we let go of the assumption that the velocity varies linearly between the wall and streamline 2. The momentum flux through the lower half of the stream 2 is estimated as some α times $\psi_2^- u_2$ where α takes into account the nonlinearity of the velocity distribution next to the wall. Also the viscous force at the lower face of stream 2, that is at the wall, is calculated from the Reynold's number along streamline 2 taking into account the wall surface roughness. Proceeding accordingly, we obtain by applying momentum balance to stream 2

$$\left[\psi_2^+\left(\frac{3u_2^+ + u_3^+}{4}\right) + \psi_2^-\alpha u_2^+\right] - \left[\psi_2^+\left(\frac{3u_2 + u_3}{4}\right) + \psi_2^-\alpha u_2\right]$$

$$= -\Delta p A_2^x + \frac{1}{2}\left[V_{2+1/2}^+(u_3^+ - u_2^+) + V_{2+1/2}(u_3 - u_2)\right]$$

$$= -\frac{1}{2}\left[V_1^+ u_2^+ + V_1 u_2\right] + F_2 \qquad (6)$$

Coefficients $V_{2+1/2}^+$ and $V_{2+1/2}$ are given by the general expression for $V_{j+1/2}$ presented earlier. For programming ease viscous forces along the wall have been expressed as $V_1 u_2$ similar to viscous forces at the other interfaces. However V_1 is defined differently from other V's. Estimations of V_1 and α for turbulent flows are discussed in Appendix B. When the flow is laminar Eq. (6) can be used for the momentum balance for stream 2 with α set equal to one half and V_1 determined by

$$V_1 = \frac{\mu_1}{z_2} A_1^s \qquad \text{(Laminar Flow)} \qquad (7)$$

where the A_1^s represents the area of the lower surface of stream 2 and μ_1 the viscosity at the wall (V_1^+ is determined similarly). Rearranging Eq. (6) we obtain

$$-u_3^+\left[-\frac{\psi_2^+}{4}+\frac{1}{2}V_{2+1/2}^+\right]+u_2^+\left[\frac{3}{4}\psi_2^+ + \alpha\psi_2^- + \frac{1}{2}V_{2+1/2}^+ + \frac{1}{2}V_1^+\right].$$

$$= -\Delta p A_2^x + u_3\left[\frac{\psi_2^+}{4}+\frac{1}{2}V_{2+1/2}\right]+u_2\left[\frac{3}{4}\psi_2^+ + \alpha\psi_2^- - \frac{1}{2}V_{2+1/2} - \frac{1}{2}V_1\right] + F_2 \qquad (8)$$

Equations (4), (5), and (8) provide N-1 equations to solve for the N-1 unknown velocities $u_2^+, u_3^+, \ldots, u_N^+$.

Equations for the unknown enthalpies h_j^+ at $x+\Delta x$ are obtained by applying conservation of energy to the control volume under consideration;

$$\text{Energy Flux Out}\Big|_{\text{at } x+\Delta x} - \text{Energy Flux In}\Big|_{\text{at } x} =$$

$$\text{Heat Added By Conduction} + \text{Work Done by Viscous Forces} +$$

$$Q_j\ (\text{Internal Heat Generated}) + W_j\ (\text{Work Done by the Body Forces}) \qquad (9)$$

The different terms in Eq. (9) can be estimated with varying degree of accuracy (and the accompanying complexity) to reflect their relative importance for the flow being simulated. One estimate for stream j is as follows:

$$\text{Energy Flux In} = \frac{\psi_j^+}{4}\left[3h_j + h_{j+1} + \frac{1}{2}(3u_j^2 + u_{j+1}^2)\right]$$

$$+ \frac{\psi_j^-}{4}\left[3h_j + h_{j-1} + \frac{1}{2}(3u_j^2 + u_{j-1}^2)\right]$$

Expression for energy flux out is obtained from the expression for the energy flux in by adding superscript + to h's and u's.

Heat added by conduction across the upper surface of the control volume

$$= \frac{1}{2} H^{+}_{j+1/2}(h^{+}_{j+1} - h^{+}_{j}) + \frac{1}{2} H_{j+1/2}(h_{j+1} - h_j)$$

where we have introduced

$$H^{+}_{j+1/2} = \frac{k^{+}_{j+1/2}}{C^{+}_{j}(z^{+}_{j+1} - z^{+}_{j})} A^{s}_{j+1/2} \quad \text{and} \quad H_{j+1/2} = \frac{k_{j+1/2}}{C_j(z_{j+1} - z_j)} A^{s}_{j+1/2} \tag{10}$$

The H terms can be expressed in terms of V by using Pr, the Prandtl number. Thus we can write

$$H_{j+1/2} = \frac{V_{j+1/2}}{Pr_{j+1/2}} \quad \text{where} \quad Pr_{j+1/2} = \frac{\mu_{j+1/2} C_j}{k_{j+1/2}} \tag{11}$$

Work done by the viscous forces on the upper surface

$$= \frac{1}{2}\left[V^{+}_{j+1/2}(u^{+}_{j+1} - u^{+}_{j})\left(\frac{u^{+}_{j+1} + u^{+}_{j}}{2}\right) + V_{j+1/2}(u_{j+1} - u_j)\left(\frac{u_{j+1} + u_j}{2}\right)\right]$$

$$= \frac{1}{4}\left[V^{+}_{j+1/2}(u^{+2}_{j+1} - u^{+2}_{j}) + V_{j+1/2}(u^{2}_{j+1} - u^{2}_{j})\right]$$

Expressions for the heat transferred and the work done across the lower surface can be written down from the corresponding one's for the upper surface by replacing $j+1$ by j, j by $j-1$, and $j+1/2$ by $j-1/2$. Substituting the various estimates into Eq. (7) and rearranging to get all the unknown h^{+}_{j}'s on the left hand side of the equation we get,

$$-h_{j+1}^{+}\left[-\frac{\psi_j^{+}}{4}+\frac{1}{2}H_{j+1/2}^{+}\right]+h_j^{+}\left[\frac{3}{4}\psi_j^{+}+\frac{3}{4}\psi_j^{-}+\frac{1}{2}H_{j+1/2}^{+}+\frac{1}{2}H_{j-1/2}^{+}\right]$$

$$-h_{j-1}^{+}\left[-\frac{\psi_j^{-}}{4}+\frac{1}{2}H_{j-1/2}^{+}\right]$$

$$=h_{j+1}\left[\frac{\psi_j^{+}}{4}+\frac{1}{2}H_{j+1/2}\right]+h_j\left[\frac{3}{4}\psi_j^{+}+\frac{3}{4}\psi_j^{-}-\frac{1}{2}H_{j+1/2}-\frac{1}{2}H_{j-1/2}\right]$$

$$+h_{j-1}\left[\frac{\psi_j^{-}}{4}+\frac{1}{2}H_{j-1/2}\right]+\frac{u_{j+1}^{+2}}{2}\left[-\frac{\psi_j^{+}}{4}+V_{j+1/2}^{+}\right]$$

$$-\frac{u_j^{+2}}{2}\left[\frac{3}{4}\psi_j^{+}+\frac{3}{4}\psi_j^{-}+V_{j+1/2}^{+}+V_{j-1/2}^{-}\right]+\frac{u_{j-1}^{+2}}{2}\left[-\frac{\psi_j^{-}}{4}+V_{j-1/2}^{+}\right]$$

$$+\frac{u_{j+1}^{2}}{2}\left[\frac{\psi_j^{+}}{4}+V_{j+1/2}\right]+\frac{u_j^{2}}{2}\left[\frac{3}{4}\psi_j^{+}+\frac{3}{4}\psi_j^{-}-V_{j+1/2}-V_{j-1/2}\right]$$

$$+\frac{u_{j-1}^{2}}{2}\left[\frac{\psi_j^{-}}{4}+V_{j-1/2}\right]+Q_j+W_j \qquad (12)$$

for $j = 3,4,\ldots,N-1$

The equation that results on applying the energy balance to stream N can be obtained from Eq. (12) by letting j equal to N and setting ψ_N^{+}, h_{n+1}^{+}, h_{n+1}, u_{N+1}^{+}, u_{N+1}, $H_{N+1/2}^{+}$, $H_{N+1/2}$, $V_{N+1/2}^{+}$, and $V_{N+1/2}$ equal to zero.

Next, following the procedure outlined for the momentum balance, we apply energy balance to stream 2 to obtain,

$$\frac{\psi_2^+}{4}\left[(3h_2^+ + h_3^+) + \frac{1}{2}(3u_2^{+2} + u_3^{+2})\right]$$

$$- \frac{\psi_2^+}{4}\left[(3h_2 + h_3) + \frac{1}{2}(3u_2^2 + u_3^2)\right]$$

$$+ \psi_2^-\left[\beta(h_2^+ - h_w^+) + \delta\frac{1}{2}u_2^{+2}\right] - \psi_2^-\left[\beta(h_2 - h_w) + \frac{1}{2}\delta u_2^2\right]$$

$$= \frac{1}{2}\left[H_{2+1/2}^+(h_3^+ - h_2^+) + H_{2+1/2}(h_3 - h_2)\right] - q_w$$

$$+ \frac{1}{4}\left[V_{2+1/2}^+(u_3^{+2} - u_2^{+2}) + V_{2+1/2}(u_3^2 - u_2^2)\right] + Q_2 + W_2 \qquad (13)$$

In Eq. (13) β and δ are correction factors to take care of nonlinear variation between wall and the streamline 2 when the flow is turbulent and are further discussed in Appendix B. Also in Eq. (13) h_w represents the enthalpy evaluated at the wall temperature and q_w the heat transfer to the wall. For flows with wall temperature specified q_w can be written as

$$q_w = \frac{1}{2}\left[H_1^+(h_2^+ - h_w^+) + H_1(h_2 - h_w)\right] \qquad (14)$$

where H_1 is calculated from V_1 via the relation given in Eq. (11). When q_w is specified Eq. (14) is used to eliminate h_w and h_w^+ appearing on the left hand side of Eq. (13). Equations (12), (13) and (14) provide N equations for the N unknowns h_N, h_{N-1},...,h_2 and either one of h_w or q_w

The proceeding equations for the calculations of u_j^+'s and h_j^+'s are to be supplemented with equations given in Appendices A and B to calculate the duct width at $x + \Delta x$ in terms of u_j^+'s and ρ_j^+'s.

In summary, Eqs. (4), (5), (8), (12), (13), (14), (B-2), (A-13) [or (B-3), (A-14), and (A-15)], along with the thermodynamic relations interrelating h, ρ, T, and p are our basic set of equations to calculate the two dimensional duct flow.

III. SOLUTION TECHNIQUE

Overall methodology adopted to solve the duct flow equations will depend upon the given design specifications, e.g. pressure gradient is specified or variation of duct width is specified, etc. In any case one has to solve the momentum and energy difference equations given in section II. The set of simultaneous equations to solve for new u_j^+, as also the set to solve for new h_j^+, are of the general form

$$-a_j X_{j+1} + b_j X_j - c_j X_{j-1} = d_j, \tag{15}$$

where X_j stands for the unknowns u_j^+ or h_j^+. Coefficients a_j, b_j and c_j due to their dependence on V_j^+ or H_j^+ are indirectly functions of X_j. Thus Eq. (15) is nonlinear. If we had approximated the viscous forces and the heat transfer across stream interfaces by their values at x rather than by their averages between x and $x + \Delta x$ coefficients a_j, b_j and c_j would have been independent of X_j. In that case Eq. (15) will constitute a set of linear tri-diagonal equations and can be solved directly by the standard algorithm to solve such equations. In the present case these equations are solved by iteration.

At first we calculate a_j, b_j, and c_j for the momentum difference equations setting V^+ equal to V and solve for new u_j^+ using the algorithm to solve directly tri-diagonal equations. With these new u_j^+ and H^+ set equal to H we calculated a_j, b_j, c_j and d_j for the energy difference equations and solve for h_j^+. Next we calculate z_j^+ using newly calculated flow properties at $x + \Delta x$. The procedure is then repeated this time around calculating V^+ and H^+ using the newly calculated values of u_j^+, z_j^+, etc. The iteration

is continued until the desired convergence is achieved. The number of required iterations increases, in regions of rapidly changing temperature and velocity profiles. These are also the regions where had we calculated a_j, b_j and c_j from V and H alone (instead V, V^+ and H, H^+) the error introduced by such approximation would have been the greatest.

When the pressure gradient is specified the z_j^+'s calculated at the end of the last iteration give the channel width at $x + \Delta x$. When the channel width is specified iteration on pressure gradient is required. The iteration in the pressure gradient can be performed either over and above the iteration on V^+ and H^+ or combined into the V^+-H^+ iteration loop. In the latter case at the end of each iteration when new z_j^+ and thus the channel width at $x + \Delta x$ is calculated the assumed pressure gradient is readjusted.

We conclude this section by indicating how the present approach can be used to calculate three-dimensional flows. In three-dimensional case one will partition the flow in the duct into three-dimensional streams. In rectangular channels the streams now instead of being plane layers as in two-dimensional case will be rectangular bars. The difference equations to calculate the flow can now be written down as in the two dimensional ca except now when calculating viscous forces acting upon the control volume and the heat conducted into it one will have to take into account four neighboring streams. The resulting equations now will be pentadiagonal and will require more computational time to solve.

APPENDIX A

In this appendix we present formulae for calculating ψ_j^+, ψ_j^-, A_j^x, A_j^s, and z_j for the rectangular and cylindrical ducts. These are the formulae we have used in our computational works and can be easily modified to fit the requirements of other problems. For cylindrical ducts we introduce r_j, the distance of streamline j from the centerline. For duct of radius R_0, r_j and z_j are related by

$$r_j = R_0 - z_j \tag{A-1}$$

For rectangular ducts the formulae are for a duct of unit width. (Formulae are presented in forms to reveal their origin. They can be further contracted or simplified for computational efficiency.)

MASS FLOW RATES

(a) Rectangular Ducts

$$\psi_j^+ = \left(\frac{3\rho_j u_j + \rho_{j+1} u_{j+1}}{4}\right)\left(\frac{z_{j+1} + z_j}{2} - z_j\right)$$

$$j = 2,3,\ldots,N-1 \tag{A-2}$$

$$\psi_j^- = \left(\frac{3\rho_j u_j + \rho_{j-1} u_{j-1}}{4}\right)\left(z_j - \frac{z_j + z_{j-1}}{2}\right)$$

$$j = 3,4,\ldots,N \tag{A-3}$$

(b) Cylindrical Ducts

$$\psi_j^+ = \left(\frac{3\rho_j u_j + \rho_{j+1} u_{j+1}}{4}\right)\pi\left[r_j^2 - \left(\frac{r_j + r_{j+1}}{2}\right)^2\right]$$

$$j = 2,3,\ldots,N-1 \qquad \text{(A-4)}$$

$$\psi_j^- = \left(\frac{3\rho_j u_j + \rho_{j-1} u_{j-1}}{4}\right)\pi\left[\left(\frac{r_j + r_{j-1}}{2}\right)^2 - r_j^2\right]$$

$$j = 3,4,\ldots,N-1 \qquad \text{(A-5)}$$

$$\psi_N^- = \left(\frac{3\rho_N u_N + \rho_{N-1} u_{N-1}}{4}\right)\pi r_{N-1}^2 \Big/ 4 \qquad \text{(A-6)}$$

Calculations of ψ_2^- are discussed in Appendix B.

CROSS-SECTIONAL AREAS

(a) Rectangular Ducts

$$A_2^x = \frac{z_3 + z_2}{2} \qquad \text{(A-7)}$$

$$A_j^x = \frac{z_{j+1} - z_{j-1}}{2} \qquad \text{(A-8)}$$

$$j = 3,4,\ldots,N-1$$

$$A_N^x = \frac{z_N - z_{N-1}}{2} \qquad \text{(A-9)}$$

(b) Cylindrical Ducts

$$A_2^x = \pi\left[r_1^2 - \left(\frac{r_2 + r_3}{2}\right)^2\right] \tag{A-10}$$

$$A_j^x = \pi\left[\left(\frac{r_j + r_{j-1}}{2}\right)^2 - \left(\frac{r_j + r_{j+1}}{2}\right)^2\right] \tag{A-11}$$

$$j = 3,4,\ldots,N-1$$

$$A_N^x = \pi r_{N-1}^2 \Big/ 4 \tag{A-12}$$

z_j AND r_j TERMS

When the ρ's and u's have been calculated at a new position along the duct the z's and r's at the new position are then calculated by the following formulae,

(a) Rectangular Ducts

After z_2 has been calculated as discussed in Appendix B we calculate $z_3, z_4,\ldots,z_N$ by

$$z_j = z_{j-1} + \frac{2(\psi_{j-1}^+ + \psi_j^-)}{\rho_j u_j + \rho_{j-1} u_{j-1}} \tag{A-13}$$

(b) Circular Ducts

$$r_{N-1} = \left[\frac{8(\psi_{N-1}^+ + \psi_N^-)}{\pi(3\rho_N u_N + 5\rho_{N-1} u_{N-1})}\right]^{1/2} \tag{A-14}$$

$$r_{j-1} = \left[\frac{4\psi^{+}_{j-1}}{\pi(3u_{j-1} + u_j)} + \frac{4\psi^{-}_{j}}{\pi(3u_j + u_{j-1})} + r_j^2 \right]^{1/2}$$

$$j = N-1,\ldots,3. \qquad \text{(A-15)}$$

Radius r_1 is calculated as discussed in Appendix B.

SURFACE AREAS

(a) Rectangular Ducts

for all interfaces $\qquad A^{s}_{j+1/2} = \Delta x \qquad \text{(A-16)}$

(b) Circular Duct

for all interfaces $\qquad A^{s}_{j+1/2} = \Delta x\, \pi\, (r_j + r_{j+1}) \qquad \text{(A-17)}$

APPENDIX B

In this appendix we will discuss how to calculate the various correction factors that take into account the non-linear nature of velocity distribution between streamline 2 and the wall. There are many approaches possible to calculate these correction factors. The discussion that follows presents a method rather than the only one. For flow through circular pipes thickness of stream 2, i.e. $z_2 - z_1$, is assumed so small compared with the duct radius that we can neglect the curvature of stream 2. In evaluating the various integrals we have held ρ constant equal to some $\bar{\rho}$. For incompressible flows $\bar{\rho}$ is, of course, just equal to ρ. For compressible flows $\bar{\rho}$ is taken to be the average of ρ between streamline 2 and the wall.

We introduce mass flux correction factor, γ, by

$$\gamma = \frac{1}{u_2 z_2} \int_0^{z_2} u \, dz \tag{B-1}$$

In terms of γ we can write

$$\psi_2^- = \gamma \bar{\rho} u_2 z_2 \tag{B-2}$$

Equation (B-2) is used at the beginning to calculate ψ_2^- and later on to calculate z_2 for rectangular ducts. For circular ducts r_1 is calculated by

$$r_1 = r_2 + \psi_2^- / \gamma u_2 z_2 \tag{B-3}$$

From their definitions we have for the correction factors α and δ,

$$\alpha \bar{\psi_2} u_2 = \int_0^{z_2} \rho u^2 \, dz \, , \quad \text{that is}$$

$$\alpha = \frac{1}{\gamma u_2^2 z_2} \int_0^{z_2} u^2 \, dz \, , \tag{B-4}$$

and similarly,

$$\delta = \frac{1}{\gamma u_2^3 z_2} \int_0^{z_3} u^3 \, dz \tag{B-5}$$

If one assumes as is often done, that the variation of h in the near wall region is similar to u then the correction factor β will be obviously equal to α. If some other variation of h is assumed then the integral

$$\beta = \frac{1}{\gamma u_2 z_2 h_2} \int_0^{z_2} hu \, dz$$

evaluated with the assumed variation of h will yield β.

The velocity distribution in the near wall region, needed to evaluate the proceeding integrals, is calculated from the relation

$$\mu_t \frac{du}{dz} = \tau_w \tag{B-6}$$

where τ_w is the shear stress at the wall and μ_t is the viscosity including the turbulent contributions. To proceed further one must select some expression for μ_t. To illustrate the procedure we use von Driest's relation [8]

for μ_t (this relation has been successfully used in our own computations [7] and by several other authors [3,4]). For smooth walls we have from [8]

$$\mu_t = \mu + \rho k^2 z^2 \left[1 - \exp(-z\sqrt{\tau_w \rho}/\mu A)\right]^2 \frac{du}{dz} \tag{B-7}$$

where μ is the laminar viscosity and k and A are constants. Introducing non-dimensional variables

$$z_* = z\sqrt{\tau_w \rho}/\mu \ , \quad \text{and} \quad u_* = \rho u/\sqrt{\tau_w \rho} \tag{B-8}$$

we obtain on integrating (B-6) using (B-7)

$$u_* = \int_0^{z_*} \frac{2dz_*}{1 + \left[1 + 4k^2 z_*^2 \{1 - \exp(-z_*/A)\}^2\right]^{\frac{1}{2}}} \tag{B-9}$$

In terms of the non-dimensional variables integrals (B-1), (B-4), and (B-5) become, omitting the subscript 2,

$$\gamma = \frac{1}{u_* z_*} \int_0^{z_*} u_* dz_* \tag{B-10}$$

$$\alpha = \frac{1}{\gamma u_*^2 z_*} \int_0^{z_*} u_*^2 dz_* \ , \quad \text{and} \tag{B-11}$$

$$\delta = \frac{1}{\gamma u_*^3 z_*} \int_0^{z_*} u_*^3 dz_* \tag{B-12}$$

Introducting friction factor, f, and Reynolds number, Re, we have in terms of non-dimensional variables

$$f = \frac{\tau_w}{\rho u^2} = \frac{1}{u_*^2} , \quad \text{and} \tag{B-13}$$

$$Re = \frac{\rho u z}{\mu} = u_* z_* . \tag{B-14}$$

After having calculated the velocity distribution from (B-9) γ, α, δ, and f can be next evaluated as functions of Re once and for all and the results then later used during the flow field computations. We have numerically integrated these integrals using $k = 0.4$ and $A = 26$, the values recommended by Van Driest [8] for smooth walls. The results of the calculations are shown in Figs. (B-1) and (B-2). The numerically obtained relation between any of the factors and Re can now be stored for later use either by fitting a high order polynomial over the whole range of Re of interest or by breaking the range of Re into small segments and then approximating the relation over each segment by a straight line. The quantity V_1 introduced in section II in Eq. (6) is in terms of f given by

$$V_1 = f \frac{\rho_1 + \rho_2}{2} u_2 \tag{B-15}$$

and V_1^+ is given similarly with the quantities in the right hand side of (B-15) evaluated at $x + \Delta x$.

APPENDIX C

In this appendix we present two sets of computations of turbulent flow in smooth pipes using the present approach. In these computations shear stress at the wall was calculated from the local Reynolds' number as discussed in Appendix B. Turbulent shear stress, μ_t, in the main body of the flow was calculated using Prandtl's mixing length hypothesis,

$$\mu_t = \rho \ell^2 \left(\frac{du}{dz}\right)^2 \tag{C-1}$$

Variation of the mean free path, ℓ, along z, the radial distance from the wall, was calculated from the equation

$$\frac{\ell}{R} = 0.14 - 0.08\left(1 - \frac{z}{R}\right)^2 - 0.06\left(1 - \frac{z}{R}\right)^4 \tag{C-2}$$

where R is the radius of the pipe. Relation given in (C-2) is obtained from the experimental data for pipes as discussed in Ref. [1].

DEVELOPING FLOW

In the first set of the computations we calculated the developing velocity profiles in the inlet region of a smooth pipe starting with slug flow (radially uniform velocity) condition at the inlet. Computations were carried out with twenty radial grid points, with the grid spacing increasing as we moved away from the wall. Axial integration step, Δx, was taken at start equal to R/256. This Δx was doubled after each integration step until it reached the value R/4; from there on

Δx was held constant. The iteration in the pressure gradient was carried out over and above the iteration on V^+ (refer to section III). The formula used to iterate on pressure gradient is discussed at the end of this appendix. The V^+ iteration was carried until the velocity next to the wall varied less than 0.02 percent, and the pressure iteration was carried out until the pipe diameter at the new x was within 0.02 precent of its specified constant value. For each integration step it took a combined total of 5 to 6 iterations to meet both V^+ and pipe diameter convergence criteria. The initial guess for pressure gradient at each new x was obtained from the previous two values using a linear relation.

The results of the computations are shown in Figs. (C-1) and (C-2). In these figures u_b represents the bulk velocity, ρ the density, r the distance from the center line, x the distance from the entrance, and D the pipe diameter. In Fig. (C-1) we have plotted the computed radial vellcity profiles at three values of x/D. Also shown in this figure are experimental values obtained by Barbin and Jones [10]. Small differences between the measured and the computed values are, we think, due to the following reason: in the computations we used for the radial variation of ε Eq. (C-2), which is for fully developed turbulent flow; in the experiment, as noted by the authors from their turbulent intensity measurements, in the inlet region the turbulent shear stresses near the center of the pipe are below their fully developed values whereas near the wall they exceed their fully developed values. In Fig. (C-2) we have shown the computed pressure drop in the entry region. Also shown in this figure are the experimental values obtained in Ref. [10].

The computed pressure gradient after about fifteen pipe diameters becomes essentially constant and is equal to the pressure gradient calculated using Moody chart [1].

FULLY DEVELOPED VELOCITY PROFILES

In the second set of computations we calculated velocity profiles for fully developed flow in a smooth pipe. These profiles were obtained by setting momentum flux out of the control volume equal to the momentum flux into the control volume. This can be accomplished easily, although not most economically with respect to computating time, by setting all the ψ_j's equal to zero in the program designed to compute developing flows. The results of the computations are shown in Fig. (C-3). The computed values are shown by discrete points to indicate the logarithmic nature of the radial grid spacing using in these computations. The non-dimensional parameters used in the figure are defined by

$$z = \frac{z\sqrt{\tau_w \rho}}{\mu} \quad \text{and} \quad u^* = \frac{\rho u}{\sqrt{\tau_w \rho}}$$

where τ_w is the shear stress at the wall. Also shown in this figure are van Driest's calculations and the relation $u^* = 5.5 + 2.5\ \ell n\, z^*$, both of which compare well with the experimental data.

PRESSURE GRADIENT CORRECTION FORMULA

As discussed in section III in the present approach when the pressure gradient along the duct is not specified it is obtained by iteration. The area of the duct at $x + \Delta x$ is calculated during the computations using a guessed Δp, the pressure drop across Δx. The guessed Δp is then

iterated upon until the calculated area at $x+\Delta x$ meets the requirements. To better guess Δp at the beginning of each new iteration a formula, based on approximate application of conservation principles, has been derived. We denote area, average density, and average velocity by A_1, ρ_1, U_1 at x, and by A_2', ρ_2', U_2' at $x+\Delta x$. Applying mass and momentum balance across Δx we obtain

$$A_1\rho_1U_1 = A_2'\rho_2'U_2' \tag{C-3a}$$

$$\dot{m}(U_2' - U_1) = -\Delta p'A_1 - F_v \tag{C-3b}$$

where $\dot{m}$ represents mass flow rate and F_v the viscous forces. Now we ask the question that to produce a small change in A_2' how much change in $\Delta p'$ is required? Let A_2, ρ_2, U_2, p_2 be another set of values at $x+\Delta x$ that also satifies Eqs. (C-3) and differ from A_2', U_2', ρ_2', p_2' by only small amounts. We set

$$A_2' = A_2 + A', \quad U_2' = U_2 + U', \quad \rho_2' = \rho_2 + \rho', \quad p_2' = p_2 + p' \tag{C-4}$$

Substituting relations from (C-4) into (C-3) we obtain after neglecting second and higher order terms in small quantities A', ρ', etc.,

$$p' = \frac{\dot{m}U_2}{A_1A_2\left(1 - \frac{\dot{m}U_2}{A_1p_2\theta}\right)}A' \tag{C-5}$$

In obtaining Eq. (C-5) we have assumed (a) the small changes in A_2', ρ_2', etc., induce negligible change in F_v, and (b) changes p' and ρ' are related by

$$\frac{p'}{p_2} = \theta \frac{\rho'}{p_2} \qquad \text{(C-6)}$$

where θ is some constant. Equation (C-5) tells, approximately, that Δp when incremented by p' will produce a change equal to A' in A_2. During computations at the end of each pressure iteration the difference between the computed and the specified area is calculated, calling it A'. Eq. (C-5) is solved for p'. The p', thus obtained, is in turn used to adjust Δp for the next iteration. In our experience so far, the most satisfactory value of θ seemed to depend on the flow being computed. In the sample calculations given in this appendix, of incompressible flow, θ was set equal to infinity.

The sample calculations were carried out using FORTRAN IV programming language on IBM 360/44 with DOS operating system. For the case of developing flow, discussed in this appendix, the program required 34 K of core memory. The computational CPU time was 2 minutes and 22 seconds to carry out the calculations to 60 pipe radii (approximately 240 integration steps).

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LIST OF FIGURES

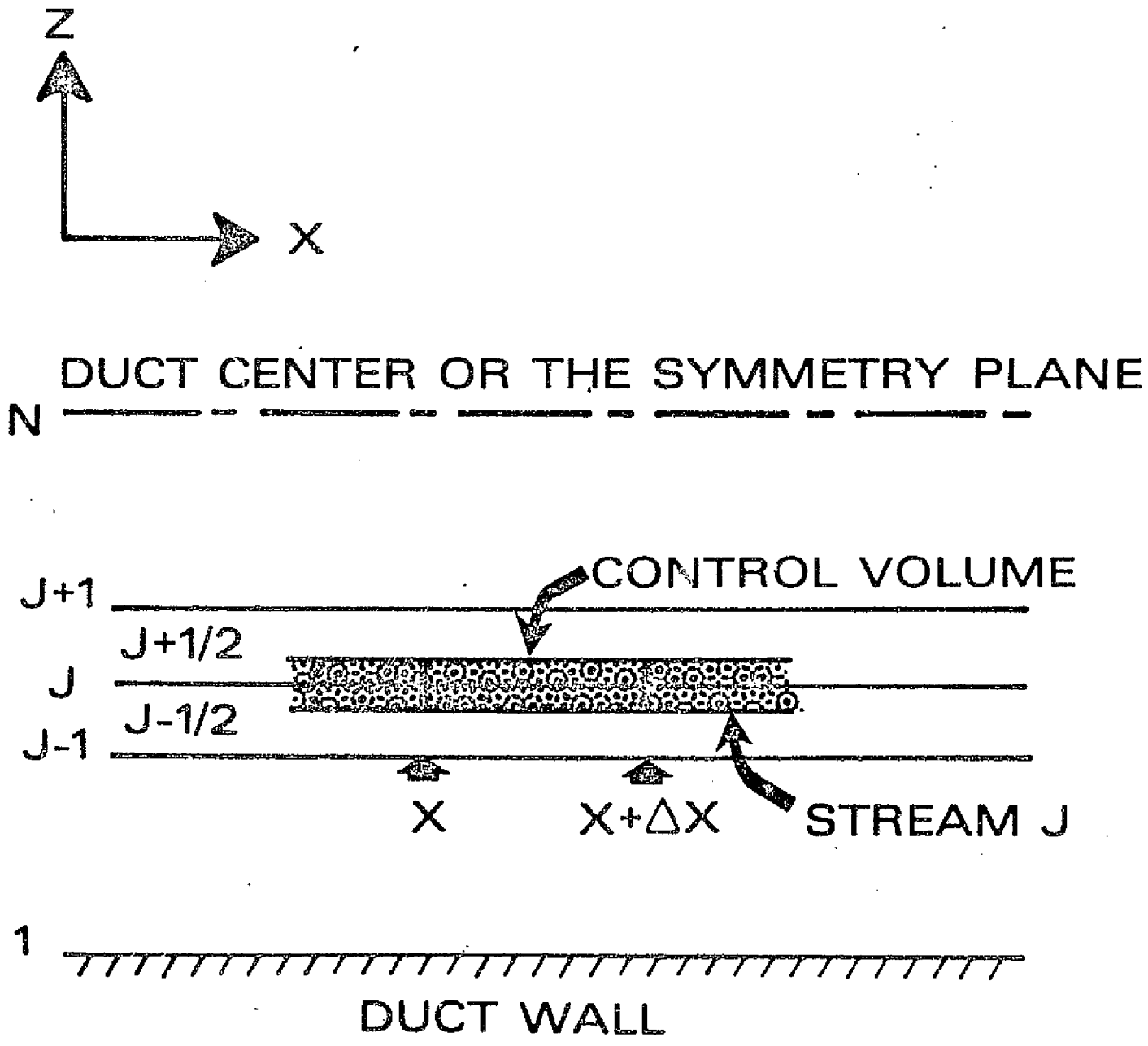
Z
X
DUCT CENTER OR THE SYMMETRY PLANE
N
CONTROL VOLUME
J+1
J+1/2
J
J-1/2
J-1
X
X+ΔX
STREAM J
1
DUCT WALL

FIG. 1

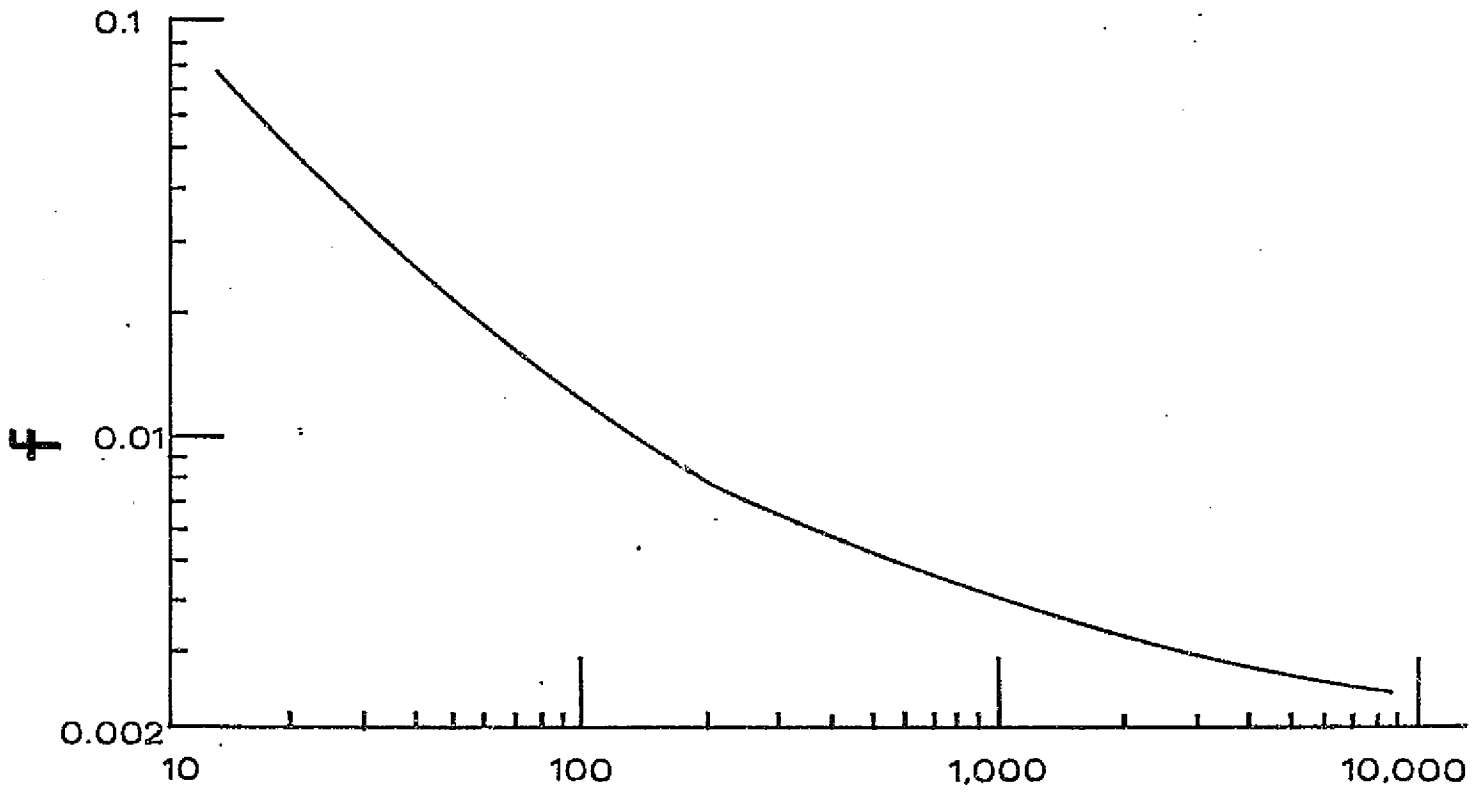

FIG. B-1

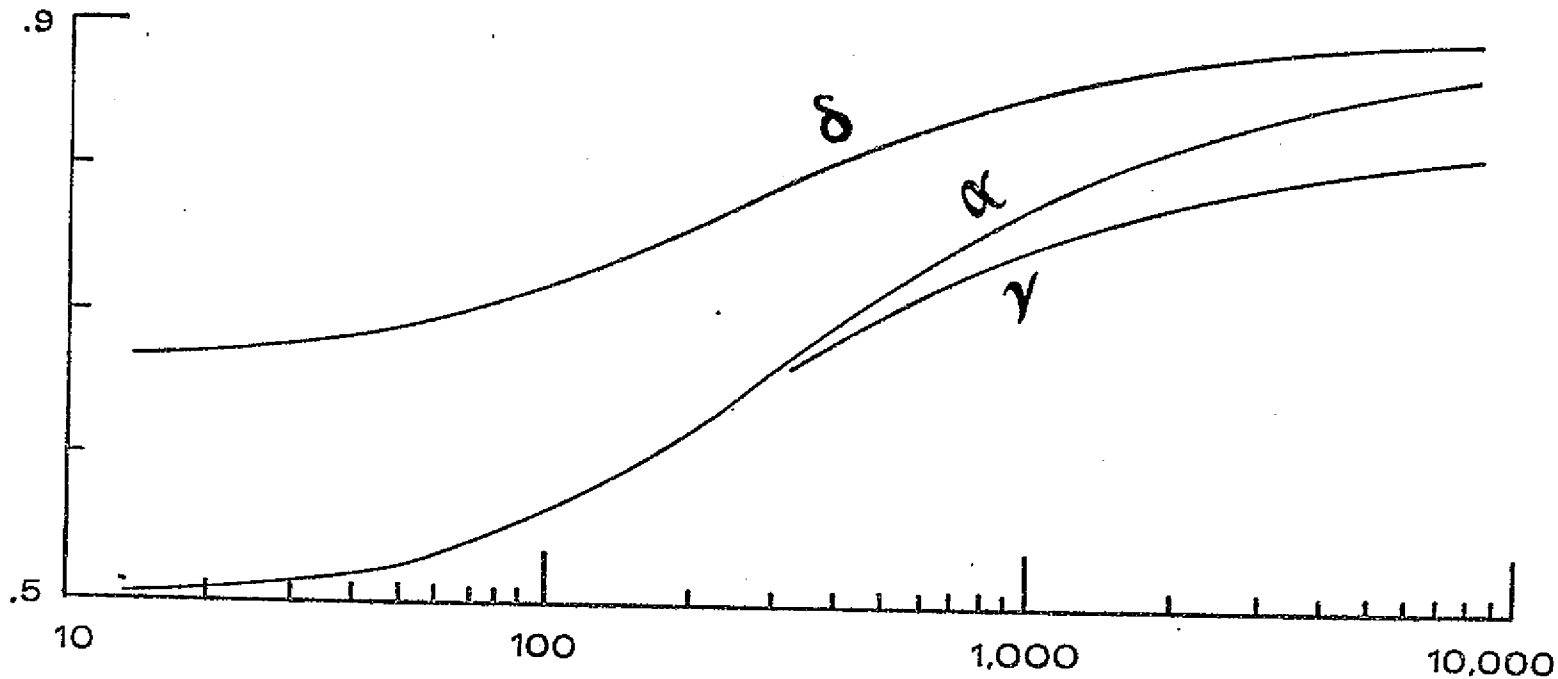

FIG. B-2

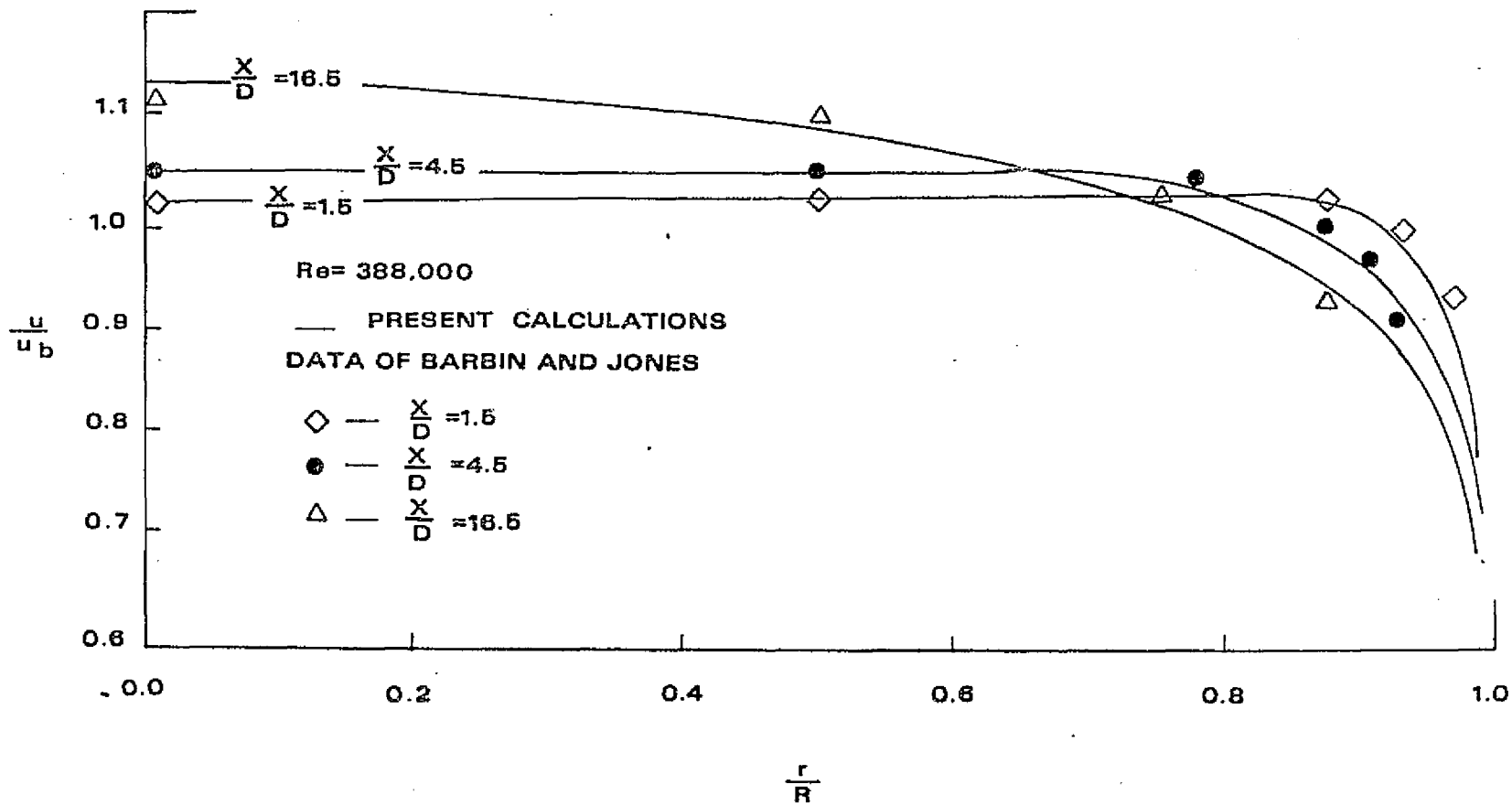
X/D =16.5
X/D =4.5
X/D =1.5
Re= 388,000
PRESENT CALCULATIONS
DATA OF BARBIN AND JONES
X/D =1.5
X/D =4.5
X/D =16.5
u/u_b
r/R
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
1.0

FIG. C-1

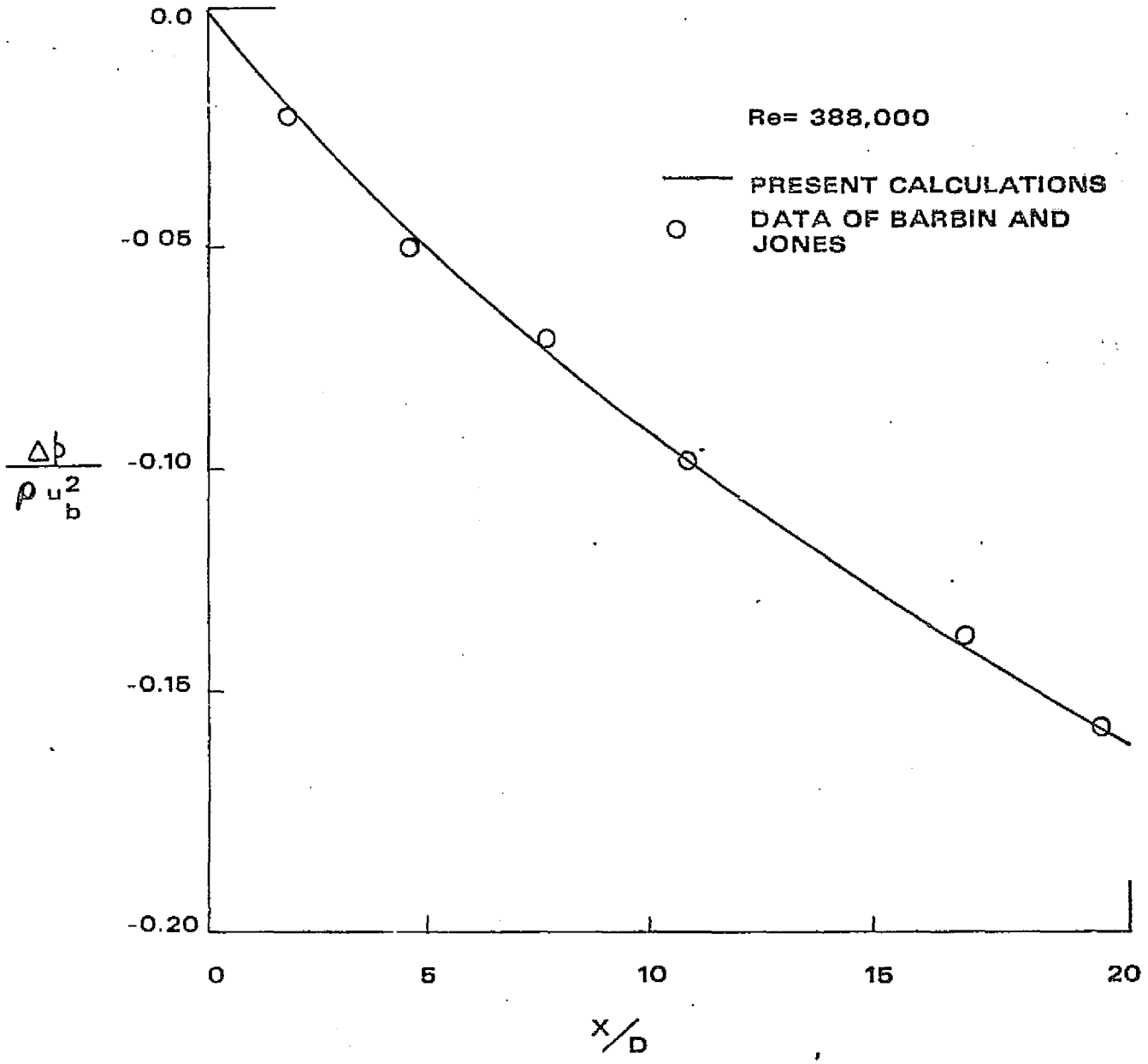
0.0
-0 05
-0.10
-0.15
-0.20
0
5
10
15
20
$\frac{\Delta p}{\rho u_b^2}$
X/D
Re= 388,000
PRESENT CALCULATIONS
DATA OF BARBIN AND JONES

FIG. C-2

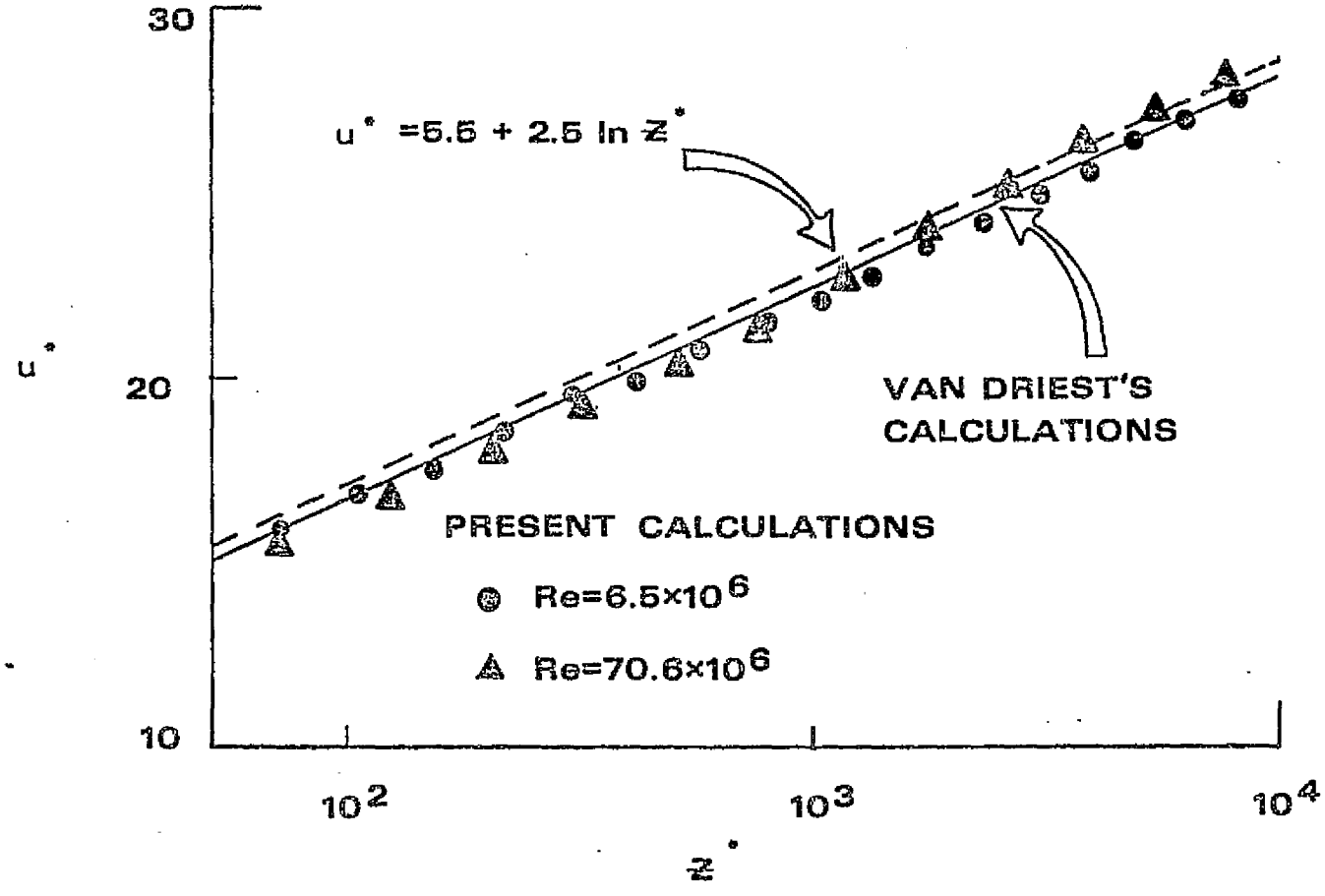
30
20
10
u*
u* =5.6 + 2.5 ln z*
VAN DRIEST'S
CALCULATIONS
PRESENT CALCULATIONS
Re=6.5×10^6
Re=70.6×10^6
10^2
10^3
10^4
z*

FIG. C-3